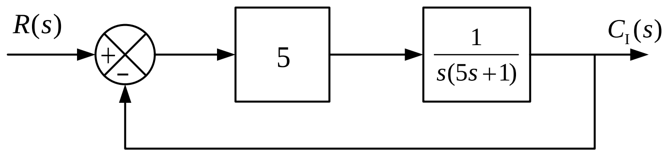
SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

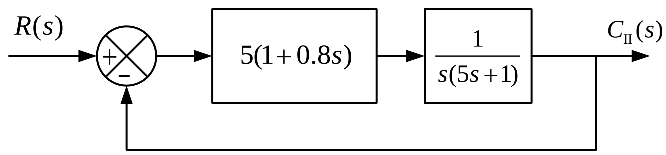
SYSTEM CONTROL **Fall 2014**

HW#3 **Assigned: September 30 (Tu)**
Due: October 14(Tu)

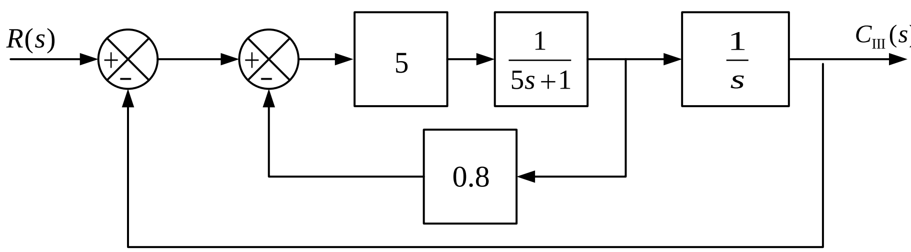
[1] Figure shows three systems. System I is a positional servo system. System II is a positional servo system with PD control action. System III is a positional servo system with velocity feedback. Compare the unit-step, unit-impulse, and unit-ramp responses of the three systems. Which system is best with respect to the speed of response and maximum overshoot in the step response?



System I



System II



System III

Figure : Positional servo system (System I), positional servo system with PD control action (System II), and positional servo system with velocity feedback (System III).

Sol)

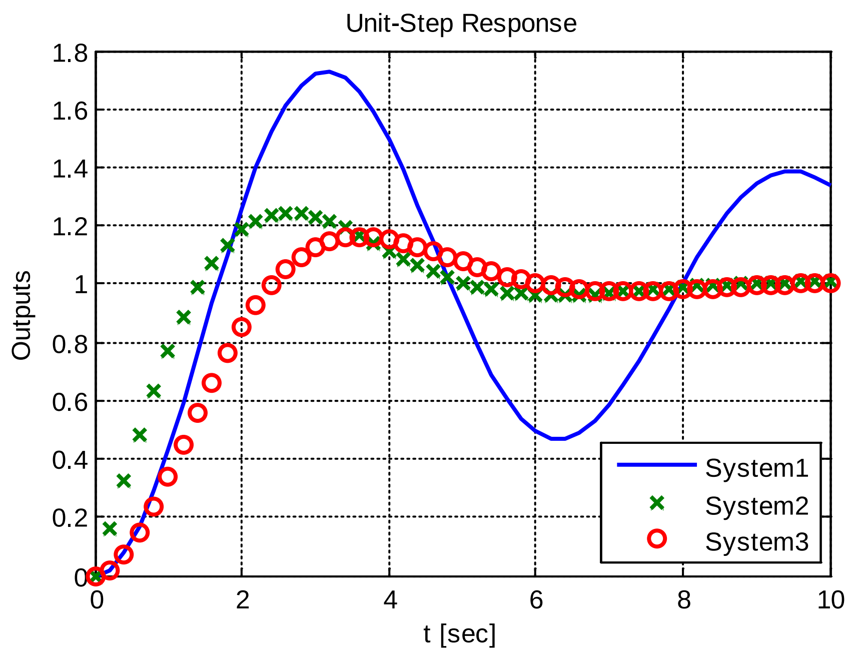
The closed-loop transfer function of system I, II, III are as follows

$$\frac{C_I(s)}{R(s)} = \frac{1}{s^2 + 0.2s + 1}$$

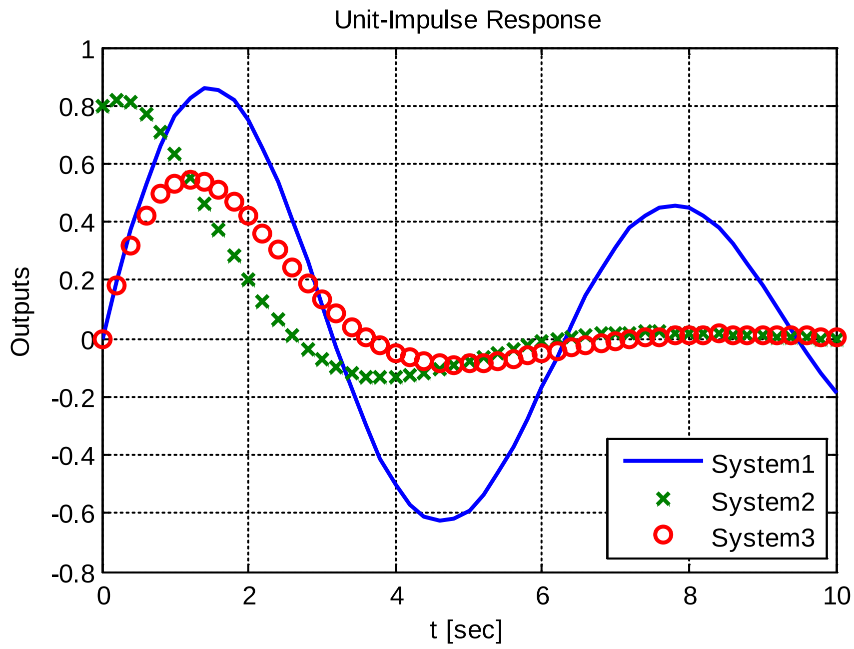
$$\frac{C_{II}(s)}{R(s)} = \frac{1 + 0.8s}{s^2 + s + 1}$$

$$\frac{C_{III}(s)}{R(s)} = \frac{1}{s^2 + s + 1}$$

The unit-step response curves for the three systems are shown in Figure 1. This figure shows that proportional-plus-derivative control action (System II) exhibits the shortest rise time. The system with velocity feedback (System III) has the least maximum overshoot, or the best relative stability, of the three systems

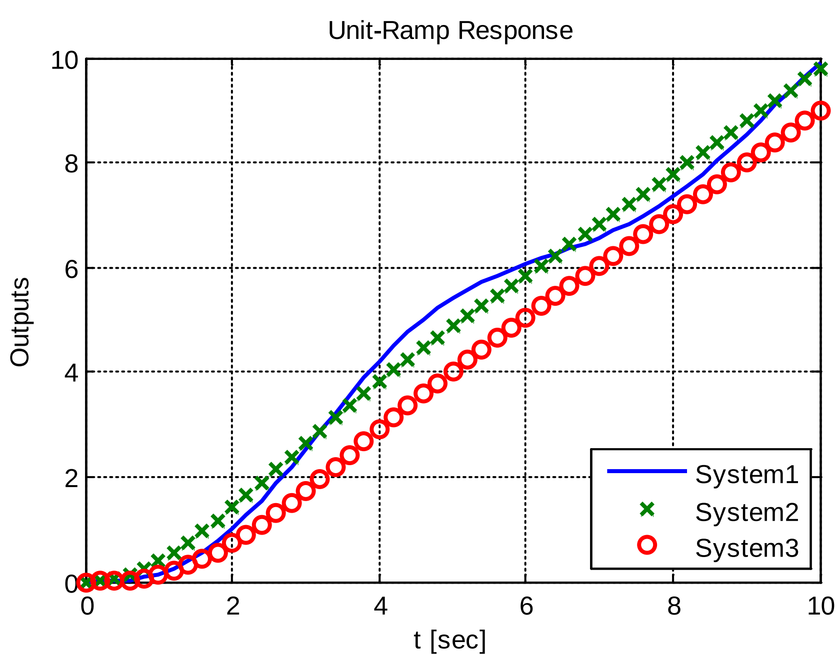


<Figure.1. Simulation result of unit-step input>



<Figure.2. Simulation result of unit-impulse input>

Figure 2 shows the unit-impulse curves for the three systems. The unit-ramp response curves for the three systems are shown in Figure 3. System II has the advantage of quicker response and less steady error in following a ramp input.



<Figure.3. Simulation result of unit-ramp input>

[2] Consider the closed-loop system shown in Figure below. Determine the range of K for stability. Assume that $K > 0$.

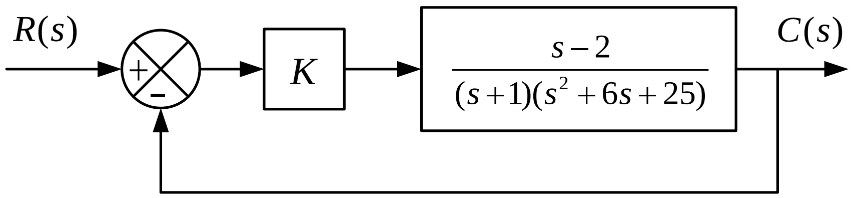


Figure: Closed-loop system.

Sol)
The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K(s-2)}{(s+1)\left(s^2+6s+25\right)+K(s-2)}$$
$$= \frac{K(s-2)}{s^3+7s^2+(31+K)+25-2K}$$

To make this system stable, above transfer function must have poles on the left-half plane. For the characteristic equation

$$s^3+7s^2+(31+K)s+25-2K=0$$

The Routh array becomes as follows :

$$\begin{array}{lcc} s^3 & 1 & 31+K \\ s^2 & 7 & 25-2K \\ s^1 & \dfrac{192+9K}{7} & 0 \\ s^0 & 25-2K & \end{array}$$

Since K is assumed to be positive, for stability, we require

$$0 < K < 12.5$$

[3] Consider a unity-feedback control system with the closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{Ks+b}{s^2+as+b}$$

Determine the open-loop transfer function *G(s)*.

Show that the steady-state error in the unit ramp response is given by

$$e_{ss} = \frac{1}{K_v} = \frac{a-K}{b}$$

Sol)

If the transfer function of open-loop is $G(s)$, closed-loop transfer function has to do with $G(s)$ as follows

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{Ks+b}{s^2+as+b}$$

Hence

$$\left(s^2+as+b\right)G(s) = (Ks+b)\left[1+G(s)\right]$$

Or

$$G(s) = \frac{Ks+b}{s(s+a-K)}$$

The steady-state error in the unit-ramp response is

$$e_{ss} = \frac{1}{K_v} = \lim_{t\to\infty} e(t) = \lim_{s\to 0}\frac{1}{sG(s)} = \lim_{s\to 0}\frac{s(s+a-K)}{s(Ks+b)} = \frac{a-K}{b}$$

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[1] B-5-13. Ogata 4^{th} ed.

[2] B-5-15.

[3] B-5-26.

[4] B-5-30.

2010, 2011

[1] B-5-15.

[2] B-5-26.

[3] B-5-30.