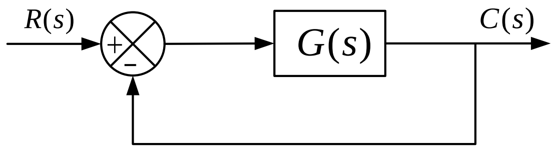
SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

SYSTEM CONTROL **Fall 2014**

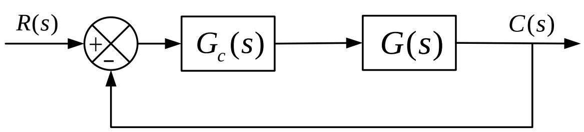
HW #4 **Assigned: October 28 (Tu)**
Due: November 4 (Tu)

1. Lead Compensator Design / Simulation



- Open-loop Transfer function : $G(s) = \dfrac{4}{s(s+2)}$
- Closed-loop Transfer function : $\dfrac{C(s)}{R(s)} = \dfrac{4}{s^2+2s+4} = \dfrac{4}{(s+1+j\sqrt{3})(s+1-j\sqrt{3})}$
- Closed-loop Poles : $s = -1 \pm j\sqrt{3}$
- Closed-loop Properties : $\zeta = 0.5, \quad \omega_n = 2 rad/s, \quad K_v = 2\sec^{-1}$

▪Compensated System



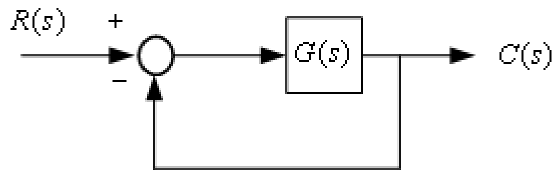
- $G_c(s) = K_c \dfrac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$: lead compensator
- Desired Properties : $\zeta = 0.5, \quad \omega_n = 4 rad/s, \quad K_v : not\ given$

1) Find $\alpha,\ T,\ K_c$.
2) Compare step input responses of the compensated system with uncompensated system using MATLAB.

Sol)

$$G(s) = \frac{4}{s(s+2)}$$

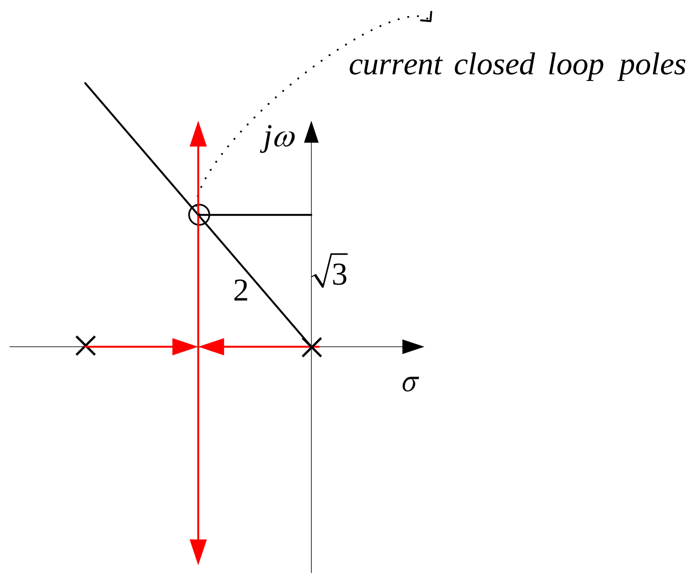
$$\frac{C(s)}{R(s)} = \frac{4}{s^2+2s+4}$$



Closed loop poles :

$$s = -1 \pm \sqrt{3}j$$

$$\begin{cases} \zeta = 0.5 \\ \omega_n = 2 \\ \text{The static velocity error constant } K_v = 2 \end{cases}$$



$$\begin{cases} E(s) = \frac{1}{1+G(s)}R(s) \\ e_{ss}(t) = \lim_{s\to 0} s\frac{1}{1+G(s)}\frac{1}{s^2} \\ \qquad = \lim_{s\to 0}\frac{1}{sG(s)} = \frac{1}{K_v} = \frac{1}{2} = 0.5 \end{cases}$$

Desired :

$$\omega_n = 4, \zeta = 0.5$$

$$\Rightarrow s = -2 \pm 2\sqrt{3}j$$

1) Angle of deficiency

$$\measuredangle \frac{4}{s(s+2)}\bigg|_{s=-2\pm 2\sqrt{3}j} = -210^\circ$$

$$\phi = 30^\circ, \ = \measuredangle K \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = 30^\circ, \quad 0<\alpha<1$$

➔ **Lead Compensator**

2) Choose $\frac{1}{T}$, $\frac{1}{\alpha T}$ such that $\measuredangle \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = 30^\circ$

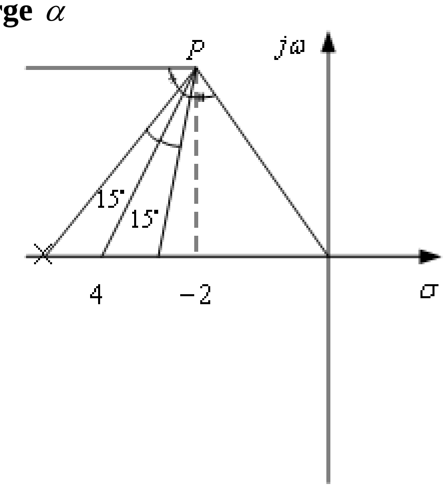
Choose α as large as possible ➔ **Increase of** K_v (Good)

Method in textbook ➔ **pole = -5.4, zero = -2.9,** α =0.536

$$T = \frac{1}{2.9} = 0.345$$

$$\alpha T = \frac{1}{5.4} = 0.185$$

Pole-zero location for large α



$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+G_c(s)G(s)} \frac{1}{s^2}$$

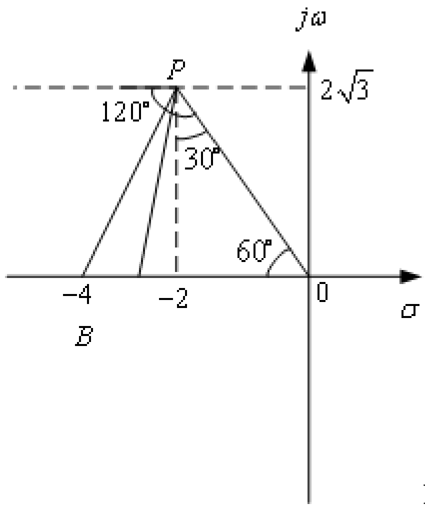
$$= \lim_{s\to 0} \frac{1}{sG_c(s)G(s)} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} sG_c(s)G(s)$$

$$= \lim_{s\to 0} K \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} \frac{4}{s+2}$$

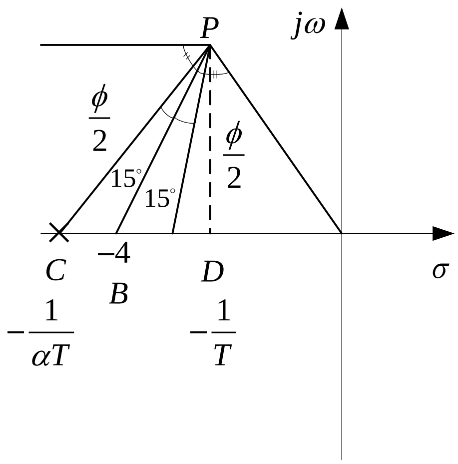
$$= K \cdot \alpha \cdot 2$$

Lead Compensator ; pole-zero selection for large α



P : desired pole $-2+2\sqrt{3}j$

Lead Compensator



C : pole : $-\dfrac{1}{\alpha T}$, **D : zero :** $-\dfrac{1}{T}$

3) Magnitude Condition

$$G_c(s)G(s) = K_c \frac{s+2.9}{s+5.4}\frac{4}{s(s+2)}$$

$$1+G_c(s)G(s)\Big|_{s=-2+2\sqrt{3}j} = 0$$

$$\left|G_c(s)G(s)\right| = 1 = K_c \left|\frac{s+2.9}{s+5.4}\right|\left|\frac{4}{s(s+2)}\right|_{s=-2+2\sqrt{3}j}$$

➔ $K_c = 4.68$

➔ **Lead Compensator**

$$G_c(s) = 4.68\frac{s+2.9}{s+5.4} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = 2.51\frac{0.345s+1}{0.185s+1}$$

[Code example]

```
%% ****** Unit-step responses ****** %%

num = [ 0 0 4 ];
den = [1 2 4];
numc = 4.68*4*[1 2.9];
denc = [1 7.4 10.8+4*4.68 4*4.68*2.9];
t = 0:0.05:5;
c1 = step(num,den,t);
c2 = step(numc,denc,t);

figure(1)
plot(t,c1,'b','linewidth',4); hold on;
plot(t,c2,'r--','linewidth',4); hold on;
grid on;
legend('uncompensated','compensated');
title('Step Response','fontsize',11);
xlabel('time[sec]','fontsize',11);
ylabel('Response','fontsize',11);
```

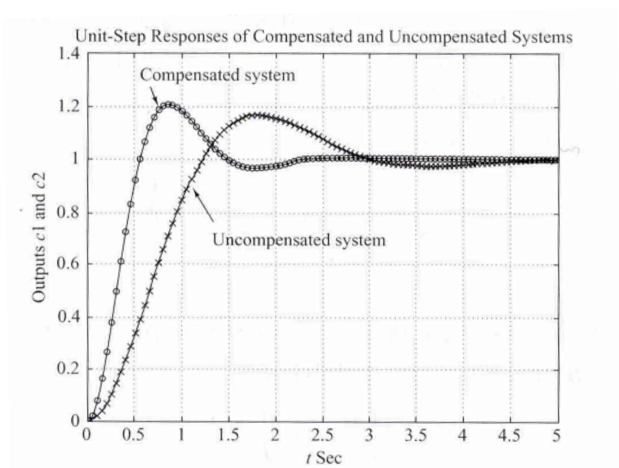
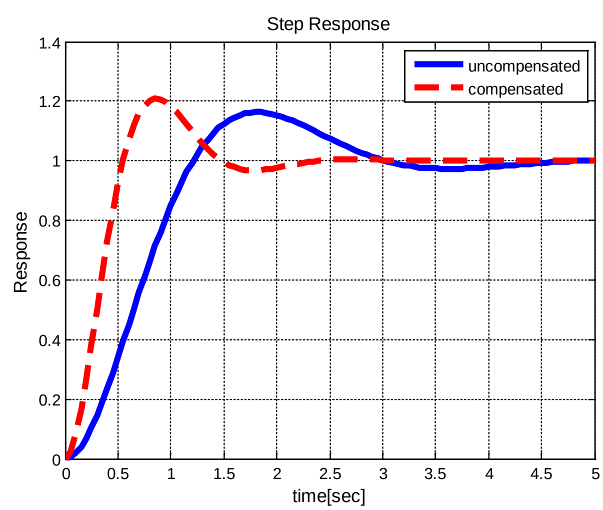
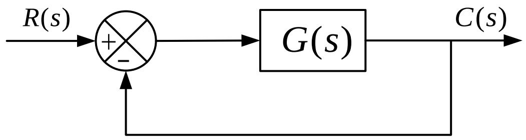


그림 7-11 보상된 시스템과 보상되지 않은 시스템의 단위계단응답



2. The transfer function of the closed-loop control system shown in the figure below is

$$G(s) = \frac{4}{s(s+0.5)}$$



Desired specifications of the compensated system are as follows:

$\zeta = 0.5, \quad \omega_n = 5 rad/s, \quad K_v = 80 \sec^{-1}$.

(1) Design a lead-lag compensator
(2) Plot root locus of the compensated and uncompensated systems.
(3) Compare step and ramp responses of the compensated and uncompensated systems.

Sol)

$$G(s)=\frac{4}{s(s+0.5)}$$

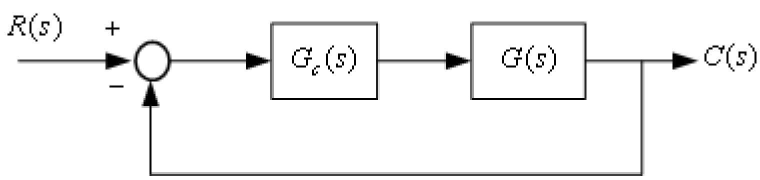
Closed loop poles :

$$s=-0.25\pm 1.9843j$$

$$\begin{pmatrix}\zeta=0.125\\ \omega_n=2rad/\sec\\ K_v=8\sec^{-1}\end{pmatrix}$$

Desired spec :

$$\begin{pmatrix}\zeta=0.5\\ \omega_n=5rad/\sec\\ K_v=80\sec^{-1}\end{pmatrix}$$



➔ **Lead – Lag compensator**

$$G_c(s)=K_c\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right),\ \gamma>1,\ \beta>1$$

Step 1. Lead Compensator

1) choose desired poles

$$s_1=-2.50\pm j4.33$$

2) angle condition

$$\measuredangle\left.\frac{4}{s(s+0.5)}\right|_{s=s_1}=-235^\circ$$

➔ **the angle deficiency = 55°**

$$\measuredangle G_c(s)+\measuredangle G(s)=-180^\circ(2k+1)$$

choose T_1 , such that pole-zero cancelation happens

$$\textbf{let} \quad s+\frac{1}{T_1}=s+0.5 \quad \Rightarrow \quad T_1=2$$

➔ **angle condition**

$$\gamma = 10.04$$

$$\measuredangle(s_1+0.5)+\measuredangle(s_1+\gamma 0.5)=55^\circ$$

3) magnitude condition

$$\left| K_c \frac{s+0.5}{s+5.021}\frac{4}{s(s+0.5)} \right|_{s=s_1} = 1$$

$$\Rightarrow K_c = 6.26$$

Step 2. Lag Compensator

1) Determine β based on desired static velocity error constant

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} sK_c\frac{\beta}{\gamma}G(s)$$

$$= \lim_{s\to 0} s(6.26)\frac{\beta}{10.04}\frac{4}{s(s+0.5)} = 4.988\beta = 80$$

$$\Rightarrow \beta = 16.04$$

2) choose T_2 , such that

i)

$$\left| \frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}} \right|_{s=s_1} \doteq 1$$

ii)

$$-5^\circ < \measuredangle \frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}} < 0$$

$$\Rightarrow T_2 \geq 5 \quad \Rightarrow let \ T_2 = 5$$

$$G_c(s) = 6.26\left(\frac{s+\frac{1}{2}}{s+\frac{10.04}{2}}\right)\left(\frac{s+\frac{1}{5}}{s+\frac{1}{16.04\times 5}}\right)$$

$$G_c(s)G(s) = \frac{25.04(s+0.2)}{s(s+5.02)(s+0.01247)}$$

[Code example]

```
%% ****** Unit-step responses ****** %%

num = [0 0 4];
den_step = [1 0.5 4];
numc = 25.04*[1 0.2];
denc_step = [1 5.02+0.01247 5.02*0.01247+25.04 25.04*0.2];
t = 0:0.05:10;
c1 = step(num,den_step,t);
c2 = step(numc,denc_step,t);

figure(1)
plot(t,c1,'b','linewidth',4); hold on;
plot(t,c2,'r--','linewidth',4); hold on;
grid on;
legend('uncompensated','compensated');
title('Unit Step Response','fontsize',11);
xlabel('time[sec]','fontsize',11);
ylabel('Response','fontsize',11);

%% ****** Unit-ramp responses ****** %%

den_ramp = [1 0.5 4 0];
denc_ramp = [1 5.02+0.01247 5.02*0.01247+25.04 25.04*0.2 0];

c3 = step(num,den_ramp,t);
c4 = step(numc,denc_ramp,t);

figure(2)
plot(t,c3,'b','linewidth',4); hold on;
plot(t,c4,'r--','linewidth',4); hold on;
grid on;
legend('uncompensated','compensated');
title('Unit Ramp Response','fontsize',11);
xlabel('time[sec]','fontsize',11);
ylabel('Response','fontsize',11);
```

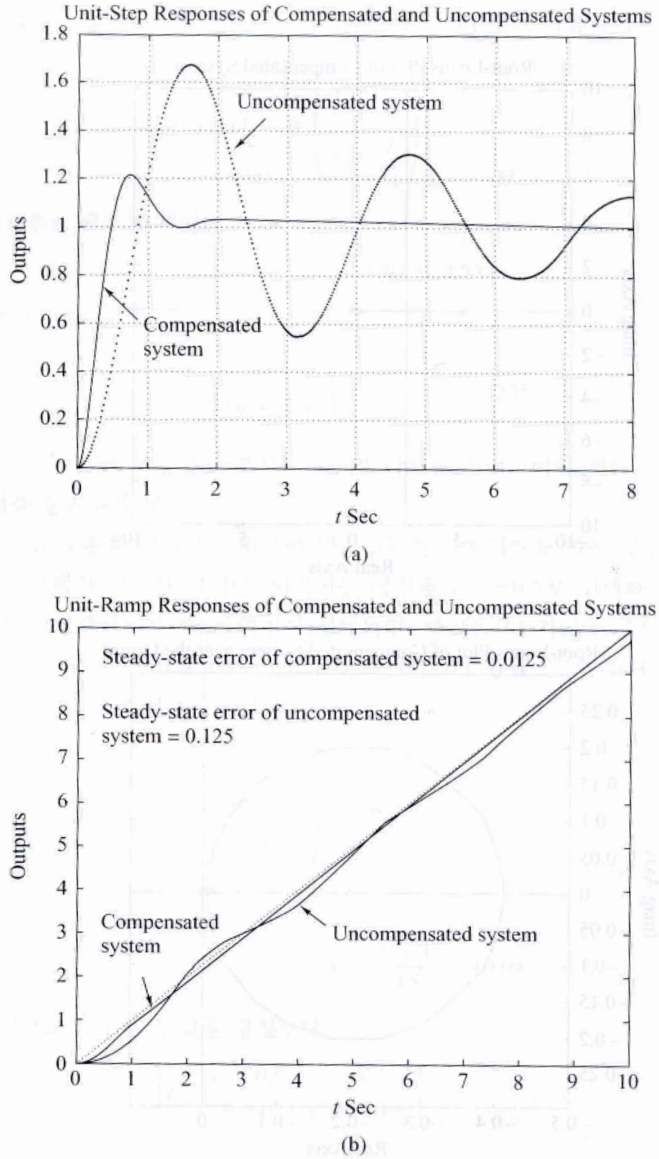
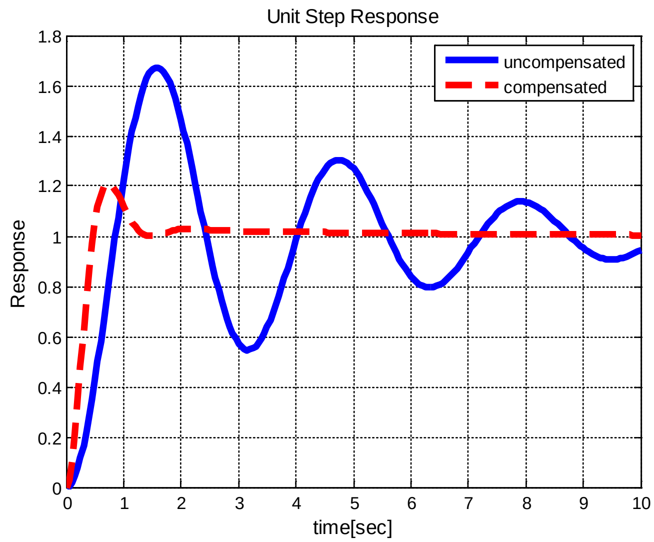
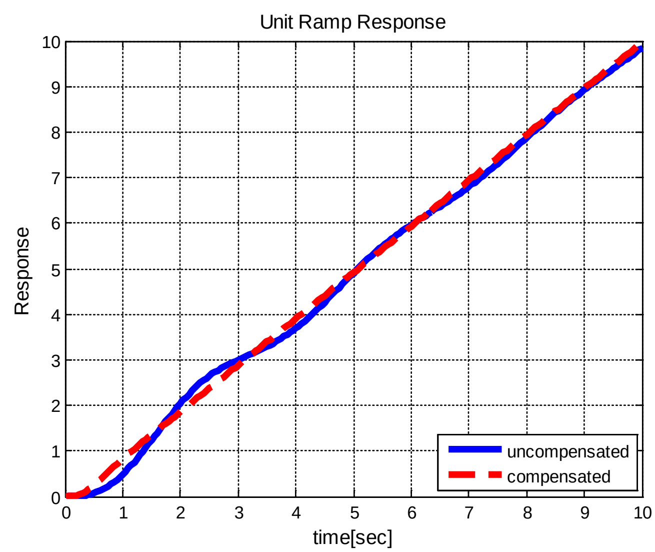


그림 7-22 보상된 시스템과 보상되지 않은 시스템의 과도응답 곡선: (a) 단위계단응답 곡선, (b) 단위램프응답 곡선.


Unit Step Response
Response
time[sec]
uncompensated
compensated
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
1
2
3
4
5
6
7
8
9
10


Unit Ramp Response
Response
time[sec]
uncompensated
compensated
0
1
2
3
4
5
6
7
8
9
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0
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