SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

SYSTEM CONTROL **Fall 2014**

HW #5 **Assigned: November 4 (Tu)**
Due: November 11 (Tu)

1. Consider the unity-feedback system with the open-loop transfer function :

$$G(s) = \frac{10}{s+1}$$

Obtain the steady-state output of the system when it is subjected to each to the following inputs:

(a) $r(t) = \sin(t + 30°)$

Sol) $C_{ss}(t) = 0.905 \sin(t + 24.8°)$

(b) $r(t) = 2\cos(2t - 45°)$

Sol) $C_{ss}(t) = 1.79 \cos(2t - 55.3°)$

(c) $r(t) = \sin(t + 30°) - 2\cos(2t - 45°)$

Sol) $C_{ss}(t) = 0.905 \sin(t + 24.8°) - 1.79 \cos(2t - 55.3°)$

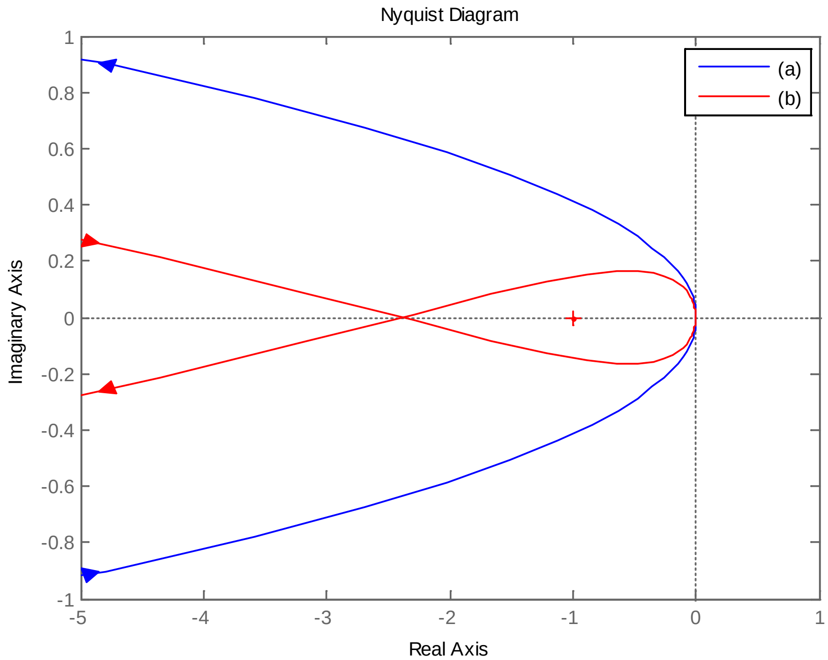
2. Sketch the polar plots of the open-loop transfer function

$$G(s)H(s) = \frac{K(T_a s + 1)(T_b s + 1)}{s^2 (Ts + 1)}$$

for the following two cases:

(a) $T_a > T > 0, \quad T_b > T > 0$
(b) $T > T_a > 0, \quad T > T_b > 0$

Sol) Typical Nyquist curves for the cases (a) and (b) are shown below.
(a) $T_a = 1.2, \; T_b = 0.5, \; T = 0.4, \; K = 0.1$
(b) $T_a = 0.6, \; T_b = 1, \; T = 2, \; K = 1$



3. Consider the closed-loop system shown in Figure 3-1. G(s) has no poles in the right-half s plane.
   If the Nyquist plot of G(s) is as shown in Figure 3-2(a), is this system stable?
   If the Nyquist plot is as shown in Figure 3-2(b), is this system stable?

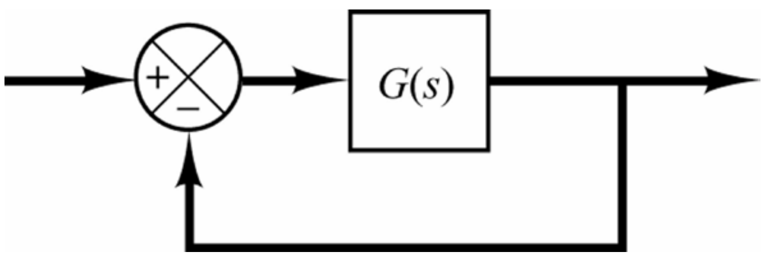


Figure 3-1 Closed-loop system

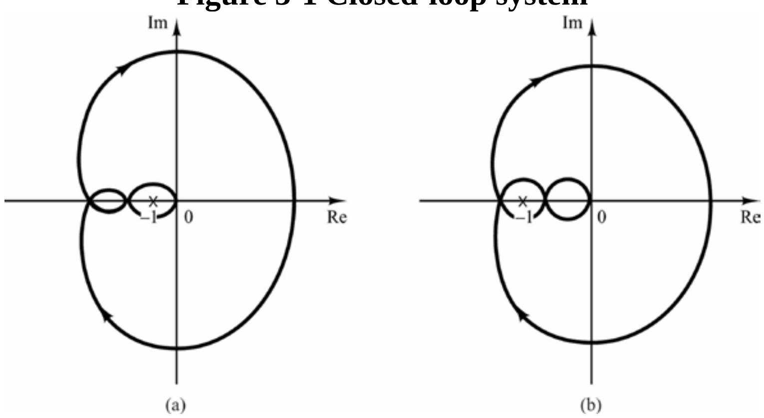


Figure 3-2 Nyquist plot

Sol) Since G(s) has no poles in the right-half s plane, the stability of the system can be studied by checking the enclosure of the -1 + j0 point by the Nyquist locus for $0 < \omega < \infty$.

If the Nyquist plot of G(s) is as shown in Figure 3-2(a), then there is no enclosure of the -1+ j0 point. Hence, the system is stable.
For the case of the Nyquist plot shown in Figure 3-2(b), the -1 + j0 point is enclosed by the Nyquist plot of G(jw) for $0 < \omega < \infty$. Hence, the system is unstable.

4. Consider the system shown in Figure 4-1. Draw a Bode diagram of the open-loop transfer function G(s). Determine the phase margin and gain margin.

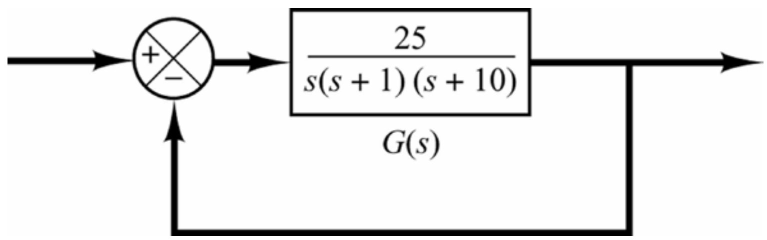
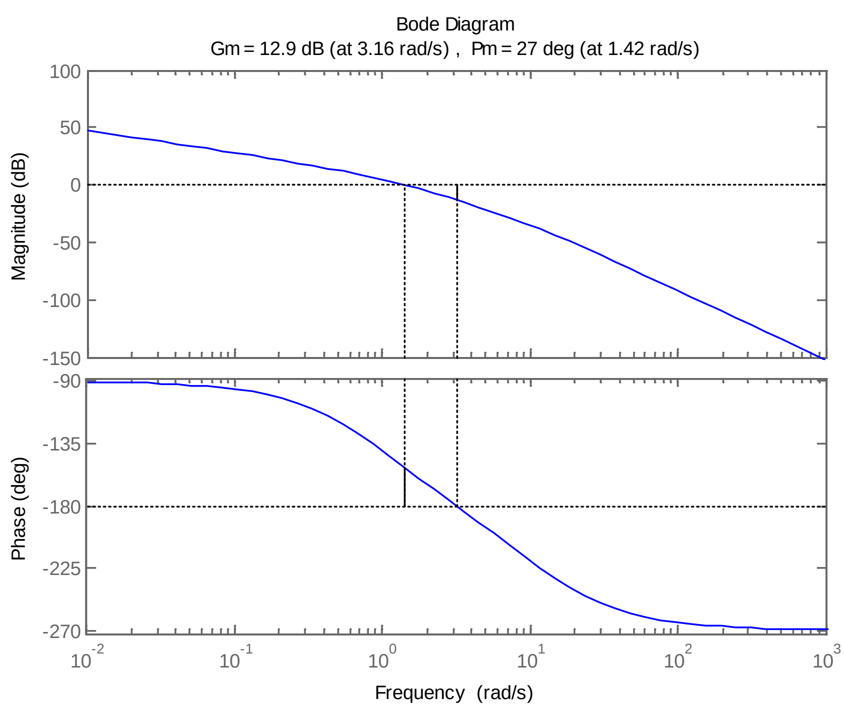


Figure 4-1 Control system

Sol) A bode diagram of the system is shown below.



From this Bode diagram, we find the phase margin and gain margin to be 27° and 13 dB, respectively.

5. Consider a unity-feedback control system with the open-loop transfer function

$$G(s) = \frac{K}{s(s^2 + s + 4)}$$

Determine the value of the gain K such that the phase margin is 50°. What is the gain margin with this gain K?

Sol) $G(s) = \dfrac{K}{s(s^2 + s + 4)} = \dfrac{0.25K}{s(0.25s^2 + 0.25s + 1)}$

The quadratic term in the denominator has the undamped natural frequency of 2 rad/sec and the damping ratio of 0.25. Define the frequency corresponding to the angle of -130° to be ω_1.

$$\angle G(j\omega_1) = -\angle j\omega_1 - \angle(1 - 0.25\omega_1^2 + j0.25\omega_1)$$
$$= -90° - \tan^{-1}\frac{0.25\omega_1}{1 - 0.25\omega_1^2} = -130°$$

Solving this last equation for ω_1, we find $\omega_1 = 1.491 rad/\sec$. Thus, the phase angle becomes equal to -130° at $\omega = 1.491 rad/\sec$. At this frequency, the magnitude must be unity, or $|G(j\omega_1)| = 1$. The required gain K can be determined from

$$|G(j1.491)| = \left|\frac{0.25K}{(j1.491)(-0.555 + j0.3725 + 1)}\right| = 0.2890K$$

Setting $|G(j1.491)| = 0.2890K = 1$, we find

$$K = 3.46$$

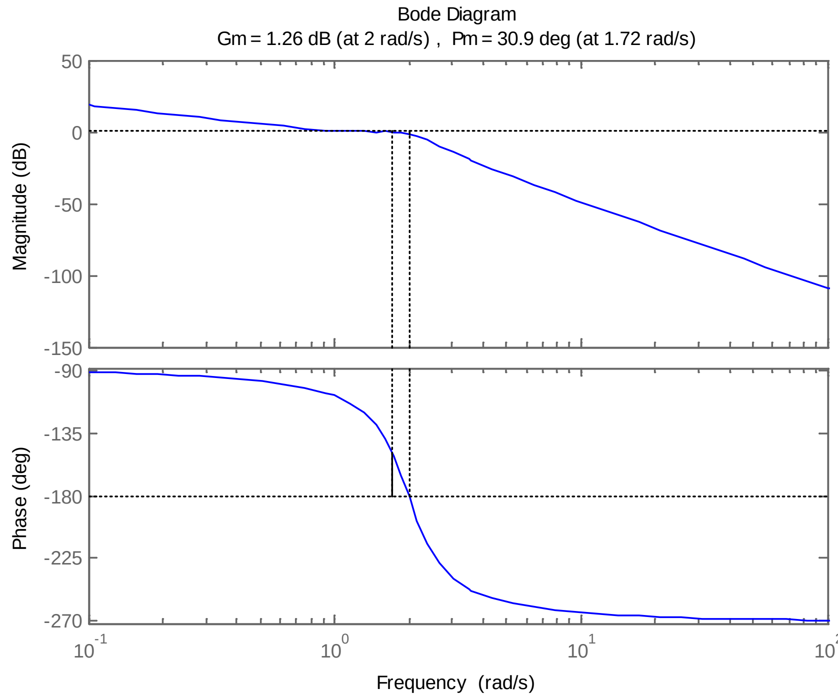
Note that the phase crossover frequency is a $\omega = 2 rad/\sec$, since

$$\angle G(j2) = -\angle(j2) - \angle(-0.25 \times 2^2 + 0.25 \times j2 + 1) = -90° - 90° = -180°$$

The magnitude $|G(j2)|$ with $K = 3.46$ becomes

$$|G(j2)| = \left|\frac{0.865}{(j2)(-1 + 0.5j + 1)}\right| = 0.865 = -1.26dB$$

Thus, the gain margin is 1.26dB. The Bode diagram of G(jw) with K=3.46 is shown below.



6. Referring to the closed-loop system shown in Figure 6-1, design a lead compensator $G_c(s)$ such that the phase margin is 45°, gain margin is not less than 8dB, and the static velocity error constant K_v is 4.0sec^{-1}. Plot unit-step and unit-ramp response curves of the compensated system with MATLAB.

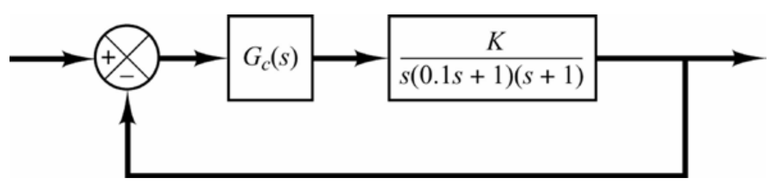


Figure 6-1 Closed-loop system

Sol) Let us use the following lead compensator:

$$G_c(s) = K_c\alpha \frac{Ts+1}{\alpha Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

Since K_v is specified as 4.0sec^{-1}, we have

$$K_v = \lim_{s\to 0} sK_c\alpha \frac{Ts+1}{\alpha Ts+1}\frac{K}{s(0.1s+1)(s+1)} = K_c\alpha K = 4$$

Let us set K=1 and define $K_c\alpha = \hat{K}$. Then,

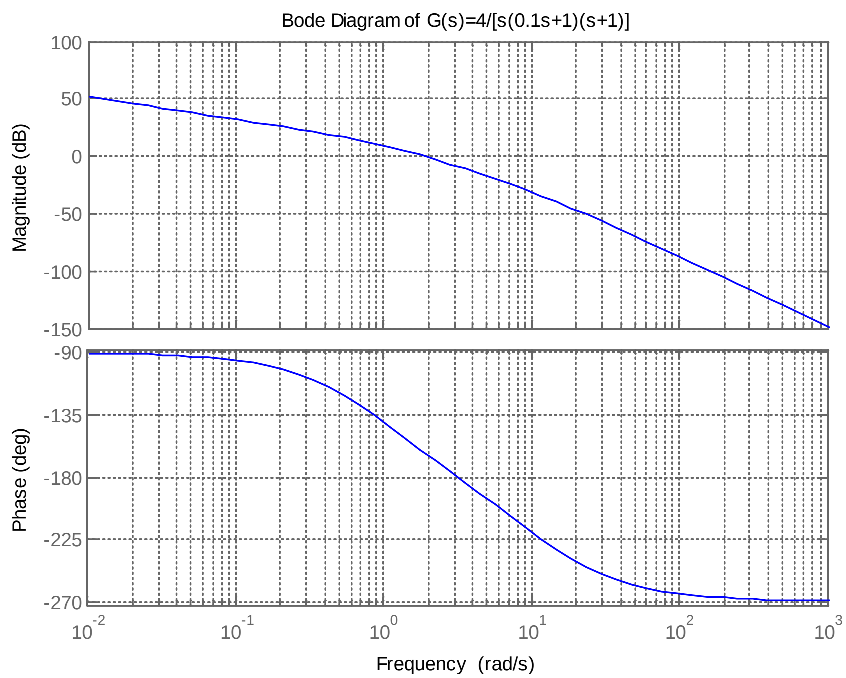
$$\hat{K} = 4$$

Next, plot a Bode diagram of

$$\frac{4}{s(0.1s+1)(s+1)} = \frac{4}{0.1s^3 + 1.1s^2 + s}$$

The following MATLAB program produces the Bode diagram shown below.

```
num = [4];
den = [0.1 1.1 1 0];
bode(num,den)
grid on
title('Bode Diagram of G(s)=4/[s(0.1s+1)(s+1)]')
```



From this plot, the phase and gain margins are 17° and 8.7dB, respectively. Since the specifications call for a phase margin 45°, let us choose

$$\phi_m = 45° - 17° + 12° = 40°$$

(This means that 12° has been added to compensate for the shift in the gain crossover frequency.) The maximum phase lead is 40°. Since

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha} \quad (\phi_m = 40°)$$

α is determined as 0.2174. Let us choose, instead of 0.2174, α to be 0.21.

Next step is to determine the corner frequencies $\omega = 1/T$ and $\omega = 1/(\alpha T)$ of the lead compensator. Note that the maximum phase-lead angle ϕ_m occurs at the geometric mean of the two corner frequencies, or $\omega = 1/(\sqrt{\alpha}T)$. The amount of the modification in the magnitude curve at $\omega = 1/(\sqrt{\alpha}T)$ due to the inclusion of the term $(Ts+1)/(\alpha Ts+1)$ is

$$\left| \frac{1 + j\omega T}{1 + j\omega\alpha T} \right|_{\omega = \frac{1}{\sqrt{\alpha}T}} = \frac{1}{\sqrt{\alpha}}$$

Note that

$$\frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{0.21}} = 2.1822 = 6.7778dB$$

We need to find the frequency point where, when the lead compensator is added, the total magnitude becomes 0 dB. The magnitude G(jw) is -6.7778 dB corresponds to $\omega = 2.81 rad/\sec$. We select this frequency to be the new gain crossover frequency ω_c. Then we obtain

$$\frac{1}{T} = \sqrt{\alpha}\omega_c = \sqrt{0.21} \times 2.81 = 1.2877$$

$$\frac{1}{\alpha T} = \frac{\omega_c}{\sqrt{\alpha}} = \frac{2.81}{\sqrt{0.21}} = 6.1319$$

Hence

$$G_c(s) = \frac{4}{0.21} \frac{s + 1.2877}{s + 6.1319} = 4 \frac{0.7766s + 1}{0.16308s + 1}$$

The open-loop transfer function becomes as

$$G_c(s)G(s) = 4 \frac{0.7766s + 1}{0.16308s + 1} \frac{1}{s(0.1s+1)(s+1)}$$

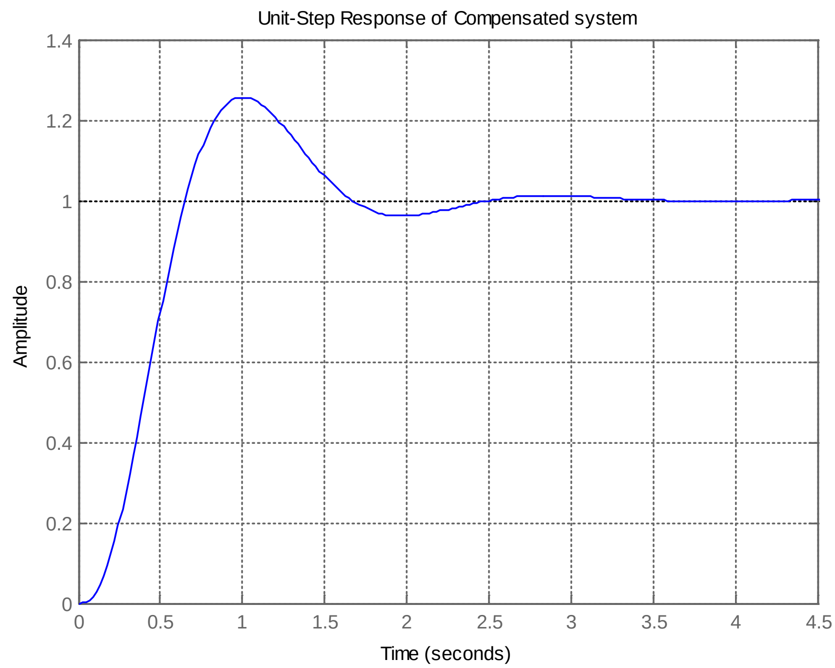
$$= \frac{3.1064s + 4}{0.01631s^4 + 0.2744s^3 + 1.2631s^2 + s}$$

This open-loop transfer function yields phase margin of 45° and gain margin of 13 dB. So, the requirements on the phase margin and gain margin are satisfied. The closed-loop transfer function of the designed system is

$$\frac{C(s)}{R(s)} = \frac{3.1064s + 4}{0.01631s^4 + 0.2794s^3 + 1.2631s^2 + 4.1064s + 4}$$

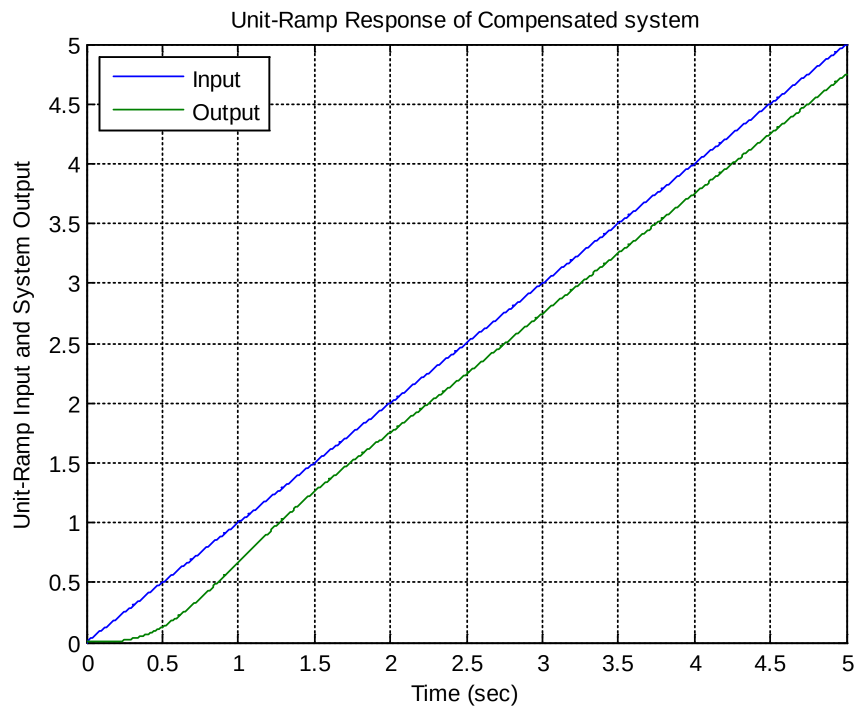
The following MATLAB program produces the unit-step response curve as shown below.

```
numc = [3.1064 4];
denc = [0.01631 0.2794 1.2631 4.1064 4];
step(numc,denc)
grid
title('Unit-Step Response of Compensated system')
```



Similarly, the following MATLAB program produces the unit-ramp response curve as shown below.

```
numc = [3.1064 4];
denc = [0.01631 0.2794 1.2631 4.1064 4 0];
t = 0:0.01:5;
c = step(numc,denc,t);
plot(t,t,t,c)
grid
legend('Input', 'Output', 'location', 'northwest')
title('Unit-Ramp Response of Compensated system')
xlabel('Time (sec)')
ylabel('Unit-Ramp Input and System Output')
```


Unit-Ramp Response of Compensated system
Input
Output
Unit-Ramp Input and System Output
Time (sec)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5