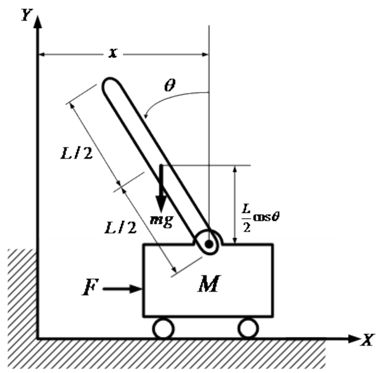
SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

SYSTEM CONTROL **Fall 2014**

HW #7 **Assigned: November 13 (Th)**
Due: December 4 (Th)

1. An inverted bar mounted on a motor-driven cart is shown in Figure below. The objective of the control problem is to transfer the position of the cart to any desired place and to keep the bar in a vertical position. The inverted bar is unstable in that it may fall over any time in any direction unless a suitable control force is applied. Here we consider only a two-dimensional problem in which the pendulum moves only in the X-Y plane. The control force F is applied at the cart. Assume that the center of gravity of the bar is at its geometric center.



(1) Obtain a mathematical model for the system.
(2) Define the state as follows:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

If we consider θ and x as the outputs of the system, then

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

Obtain a state-space model of the system. The model is of nonlinear dynamics. And then obtain a linear state-space model assuming that the θ is small.

(3) Consider force controllers of the following forms:

$$u(t) = -k \cdot \theta(t)$$

$$u(t) = -k_1 \cdot \theta(t) - k_2 \cdot \dot{\theta}(t)$$

Analyze the behavior of the system for the initial condition given as

$$x(0) = \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(4) Design a controller such that the output tracks the desired value given as

$$y_{des} = \begin{bmatrix} \theta_{des} \\ x_{des} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \end{bmatrix}$$

The initial condition is $x^T(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$

(5) Design a controller such that the output tracks the desired value given as

$$y_{des} = \begin{bmatrix} \theta_{des} \\ x_{des} \end{bmatrix} = \begin{bmatrix} 0 \\ \sin(\omega t) \end{bmatrix}$$

The initial condition is $x^T(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$

(6) For the controllers in the problem (3), (4) and (5), conduct simulations. For the problem (5), investigate the effect of ω, i.e., conduct simulations for various values of ω, and observe what happens.
For the simulation study, you choose all the parameters of the systems.