Homework #1 - Solutions

Released: 09/22/2014 (Mon) - Due: 10/01/2014 (Wed), in class

The homework will be NOT graded, but we will check for MISSING ANSWERS and CHEATING. Note that a cheated homework will get 80% of the lowest score in the class. You can give the answers <u>either</u> in English <u>or</u> Korean.

1. [Introduction to environmental engineering] Considering the history of environmental engineering, discuss why the Environmental Engineering Program exists as a module of the Department of Civil and Environmental Engineering in many colleges including us.

Answer) Environmental engineering stems from civil engineering. At the time people began to study environment issues, which can be dated back to mid-1800s, it was called as sanitary engineering with a major focus on providing safe water and removing waste generated by humans. It was mostly civil engineers who studied the sanitary engineering, and thus, sanitary engineering was considered as a part of civil engineering. As providing safe water and removing wastes involves the design and construction of pipelines, sewers, treatment facilities, and landfills, these conventional topics of environmental engineering (or sanitary engineering) are closely related to civil engineering.

2. [Basic chemistry concepts] A Na_2CO_3 solution is prepared by adding 20.00 g of Na_2CO_3 to a flask and adding pure water until it reaches the 1.00-L mark. What is the concentration of Na_2CO_3 in units of (a) mg/L; (b) molarity; and (c) normality?

Answer)

(a) $\frac{20.00\,g}{1\,L} \times \frac{10^3\,mg}{1\,g} = 2.000 \times 10^4\,mg/L$

(b) Na_2CO_3 molecular weight = 105.99

$$\frac{20.00\,g}{1\,L} \times \frac{1}{105.99\,g/mol} = 0.1887\,M$$

(c) 1 mole of Na_2CO_3 contains 2 moles of Na^+ and 1 moles of CO_3^{2-}, so each ion will have 2 moles of charge equivalents.

$$0.1887\,M \times 2 = 0.3774\,N$$

3. [Basic chemistry concepts] A $FePO_4$ solution is prepared by adding 2.4 g of $FePO_4$ to a flask and bringing the final volume to 1.00 L by adding water having a PO_4^{3-} concentration of 1.0 mg/L. What is the concentration of soluble iron in this solution? (temperature = 25˚C)

Answer)

Atomic weights: Fe 55.85, P 30.97, O 16.00

pK_s of $FePO_4$ = 21.9, $K_s = 10^{-21.9}$ *(at 25˚C)*

It looks like the solubility of $FePO_4$ is pretty low, so let's assume that only a very small fraction of the added $FePO_4$ dissolves in water.

Then, the PO_4^{3-} concentration should still be quite close to 1.0 mg/L

$$\left[PO_4^{\ 3-}\right] = \frac{1.0\,mg/L}{(30.97+4\times 16.00)\,g/mol}\frac{1\,g}{10^3\,mg} = 1.05\times 10^{-5}\,M$$

As $[PO_4^{3-}]$ is much lower than 10^{-2} M, we can further assume that (molarity) ≈ (activity)

$$K_s = 10^{-21.9} = \left[Fe^{3+}\right]\left[PO_4^{\ 3-}\right] = \left[Fe^{3+}\right]\times\left(1.05\times 10^{-5}\,M\right)$$

$$\left[Fe^{3+}\right] = 1.20\times 10^{-17}\,M$$

This proves that our assumption was correct.

4. [Basic chemistry concepts] An acetic acid (CH_3COOH) solution is prepared in water by adding **0.2** g of CH_3COONa and bringing the volume to 1.0 L with pure water. The final pH is measured to be 5.25. What are the concentrations of CH_3COO^- and CH_3COOH in solution? (temperature = 25˚C)

* Instructor note: Let me correct the amount of CH_3COONa added to the solution – **1.1 g** (in the homework I handed out) to **0.2 g**. I did not intend to make the calculation of activity coefficients necessary. Addition of 1.1 g CH_3COONa into 1.0 L water gives 0.0134 M, so in this case you cannot use the dilute solution assumption (i.e., molarity = activity). As you see below, addition of 0.2 g CH_3COONa gives 2.44 x 10^{-3} M, and you can use molarity for activity. Remember that in this class, we assume molarity = activity if the ionic strength is smaller than 10^{-2} M.

Answer)

Atomic weights: C 12.01, H 1.01, O 16.00, Na 22.99

CH_3COONa added = $\dfrac{0.2\,g}{(2\times 12.01+3\times 1.01+2\times 16.00+22.99)} = 2.44\times 10^{-3}$ *mole*

The CH_3COONa added will be dissociated into CH_3COO^- and Na^+. CH_3COO^- (acetate) may maintain its form or get a proton (H^+) to become CH_3COOH (acetic acid).

So, find that this reaction is involved:

$$CH_3COOH = CH_3COO^- + H^+$$

pK_a of CH_3COOH: 4.75

$$K_a = 10^{-4.75} = \frac{[CH_3COO^-][H^+]}{[CH_3COOH]} \qquad (1)$$

As 2.44 x 10^{-3} mole of CH_3COONa was added to 1.0 L water,

$$[CH_3COO^-] + [CH_3COOH] = 2.44\times 10^{-3}\ M \qquad (2)$$

From (1), as pH=5.25, $[H^+] = 10^{-5.25}$ *M (activity ≈ molarity as ionic strength < 10^{-2} M)*

$$[CH_3COO^-] = 3.16[CH_3COOH] \qquad (3)$$

Plugging in (3) in (2):

$$[CH_3COOH] = 5.87\times 10^{-4}\ M$$

$$[CH_3COO^-] = 1.85\times 10^{-3}\ M$$

5. [Basic chemistry concepts] Hypochlorous acid (HOCl) decays in the presence of ultraviolet radiation. Assuming that degradation occurs according to first-order kinetics, how long does it take for the concentration of hypochlorous acid to reach 0.05 mg/L if the initial concentration was 3.65 mg/L? The first-order reaction constant is 0.12 day^{-1}.

Answer) $\dfrac{d[HOCl]}{dt} = -k[HOCl]$

$$\frac{1}{[HOCl]}d[HOCl] = -k\cdot dt$$

$$\int_{[HOCl]_{t=0}}^{[HOCl]_{t=t}} \frac{1}{[HOCl]} d[HOCl] = -\int_0^t k \cdot dt$$

$$ln[HOCl]\Big|_{[HOCl]_{t=0}}^{[HOCl]_{t=t}} = -kt\Big|_0^t$$

$$ln\frac{[HOCl]_{t=t}}{[HOCl]_{t=0}} = -kt$$

$$t = -\frac{1}{k} ln\frac{[HOCl]_{t=t}}{[HOCl]_{t=0}}$$

$$Now,\ \frac{[HOCl]_{t=t}}{[HOCl]_{t=0}} = \frac{0.05\,mg/L}{3.65\,mg/L},\ k = 0.12\,day^{-1}$$

$$\therefore t = 35.8\,days$$

6. [Basic chemistry concepts] You bought a bottle of carbonated water which is completely sealed and does not have any headspace[1]. You found that the pH of the water was 3.5. You wanted to know better about the water you would drink and called the factory to find that they blow in 0.1 g of CO_2 into a carbonate-free[2] water to make a 250 mL bottle. The bottle was pressurized that all CO_2 blown in was dissolved in the water. Calculate the concentrations of [H_2CO_3*], [HCO_3^-], and [CO_3^{2-}] in the bottle. (temperature = 25˚C)

[1]This means the water is filled to the top

[2]This means that before CO_2 was blown in, there was no carbonate species ([H_2CO_3*], [HCO_3^-], or [CO_3^{2-}]) in the water.

Answer)

This is a closed system.

Carbonate system: $H_2CO_3{*} = H^+ + HCO_3^- = 2H^+ + CO_3^{2-}$

$$K_{a1} = \frac{[H^+][HCO_3^-]}{[H_2CO_3{*}]} = 10^{-6.35} \qquad (1)$$

$$K_{a2} = \frac{[H^+][CO_3^{2-}]}{[HCO_3^-]} = 10^{-10.33} \qquad (2)$$

For a closed system: $C_T = [H_2CO_3{*}] + [HCO_3^-] + [CO_3^{2-}]$

0.1 g CO_2 is added (molecular weight of CO_2 = 44.01)

$$C_T = \frac{0.1\,g/(44.01\,g/mol)}{0.25\,L} = 9.09\times10^{-3}\,M$$

At pH < pK_{a1}, $[H_2CO_3*] \approx C_T$ *(check the lecture notes!)*

$$[H_2CO_3*] = 9.09\times10^{-3}\,M$$

As pH = 3.5, $[H^+] = 10^{-3.5}\,M$ *(activity ≈ molarity for dilute solutions)*

Applying those values in Eq. (1),

$$[HCO_3^-] = 1.28\times10^{-5}\,M$$

Applying the values in Eq. (2),

$$[CO_3^{2-}] = 1.89\times10^{-12}\,M$$

7. [Basic chemistry concepts] You placed water in a bowl until it equilibrates with atmospheric CO_2 ($P_{CO2} = 10^{-3.5}$ atm). The pH of water is 7.5 and the temperature is 25˚C.

a. Calculate the concentrations of [H_2CO_3*], [HCO_3^-], and [CO_3^{2-}].

Answer)

This is an open system.

From the lecture notes,

$$[H_2CO_3*] = K_H P_{CO_2} = (10^{-3.5}\,atm)(10^{-1.5}\,M/atm) = 10^{-5}\,M$$

$$log[HCO_3^-] = -11.35 + pH = -3.85$$

$$[HCO_3^-] = 10^{-3.85}\,M = 1.41\times10^{-4}\,M$$

$$log[CO_3^{2-}] = -21.68 + 2pH = -6.68$$

$$[CO_3^{2-}] = 10^{-6.68}\,M = 2.09\times10^{-7}\,M$$

b. Calculate the carbonate alkalinity of the water.

Answer)

$$Carbonate\ \ Alkalinity = \left[HCO_3^{\ -}\right] + 2\left[CO_3^{\ 2-}\right] = \left(1.41\times10^{-4}\ M\right) + 2\cdot\left(2.09\times10^{-7}\ M\right) = 1.41\times10^{-4}\ M$$

(carbonate (CO_3^{2-}) contributes very little)

**** Note that, by convention, alkalinity is expressed in units of "mg as $CaCO_3$ / L".*

This unit indicates what concentration of $CaCO_3$ (in mg/L) does the alkalinity correspond to.

For this class, just remember that, you have to multiply your results in Molarity by "5x10^4 mg as $CaCO_3$/mol"

Applying this logic,

$$Carbonate\ \ Alkalinity = 1.41\times10^{-4}\ M \times \left(5\times10^{4}\ mg\ as\ CaCO_3\ /\ mol\right) = 70.5\ mg\ as\ CaCO_3\ /\ L$$

8. [Basic biology concepts] What are the three constituents of a nucleotide?

Answer) A phosphate, a sugar, and a base.

9. [Basic biology concepts] List the four classes of macromolecules essential for life. Briefly explain the importance of each.

Answer)

Carbohydrates: energy for life

Nucleic acids: DNA and RNA – store and transmit genetic information

Proteins: regulate cellular functions by catalyzing biochemical reactions (enzymes); support structure of organisms.

Lipids: store energy (fats), make up cell membranes (phospholipids), and used for signaling (steroids)