1. Express n and k in terms of ε and σ (or ε_1 and ε_2) by using $\varepsilon = n^2 - k^2$ and $\sigma = 4\pi\varepsilon_0 nk\nu$. (Compare with (10.15) and (10.16).)

$$\begin{cases} \varepsilon = n^2 - k^2 \rightarrow n^2 = k^2 + \varepsilon \\ \sigma = 4\pi\varepsilon_0 nk\nu \rightarrow k = \sigma / 4\pi\varepsilon_0 n\nu \end{cases}$$ 대입. $n^2 = \left(\frac{\sigma}{4\pi\varepsilon_0 n\nu}\right)^2 + \varepsilon$ 양변에 n^2을 곱하면

$$n^4 - \varepsilon n^2 - \left(\frac{\sigma}{4\pi\varepsilon_0\nu}\right)^2 = 0 \rightarrow n^2 = \frac{1}{2}\varepsilon \pm \frac{1}{2}\sqrt{\varepsilon^2 + 4\left(\frac{\sigma}{4\pi\varepsilon_0\nu}\right)^2}$$

n은 실수 이므로 $n^2 > 0$ 이다. $\therefore n^2 = \frac{1}{2}\left(\varepsilon + \sqrt{\varepsilon^2 + \left(\frac{\sigma}{2\pi\varepsilon_0\nu}\right)^2}\right)$ ((10.15)와 동일)

$$\therefore n = \pm\left[\frac{1}{2}\left(\varepsilon + \sqrt{\varepsilon^2 + \left(\frac{\sigma}{2\pi\varepsilon_0\nu}\right)^2}\right)\right]^{1/2}$$

n과 반대로, $k^2 = n^2 - \varepsilon$

$$\therefore k^4 + \varepsilon k^2 - \left(\frac{\sigma}{4\pi\varepsilon_0\nu}\right)^2 = 0 \rightarrow k^2 = -\frac{1}{2}\varepsilon \pm \frac{1}{2}\sqrt{\varepsilon^2 + 4\left(\frac{\sigma}{4\pi\varepsilon_0\nu}\right)^2}$$

$k^2 > 0$ 이므로 $k^2 = \frac{1}{2}\left(-\varepsilon + \sqrt{\varepsilon^2 + \left(\frac{\sigma}{2\pi\varepsilon_0\nu}\right)^2}\right)$ ((10.16)과 동일)

$$\therefore k = \pm\left[\frac{1}{2}\left(-\varepsilon + \sqrt{\varepsilon^2 + \left(\frac{\sigma}{2\pi\varepsilon_0\nu}\right)^2}\right)\right]^{\frac{1}{2}}$$

2. Calculate the reflectivity of silver and compare it with the reflectivity of flint glass (n = 1.59). Use λ = 0.6 μm

$R = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2}$. $R_{Ag} = \frac{(0.05-1)^2 + 4.09^2}{(0.05+1)^2 + 4.09^2} = 0.989$ (Table 10.2)

flint glass : $k \ll 1$, $\longrightarrow R_{fg} = \frac{(n-1)^2}{(n+1)^2} = \frac{(1.59-1)^2}{(1.59+1)^2} = 0.0519$

$\therefore R_{Ag} > R_{flint\ glass}$ ($\lambda = 0.6\mu m$)

3. The transmissivity of a piece of glass of thickness d = 1 cm was measured at λ = 589 nm to be 89 %. What would the transmissivity of this glass be if the thickness were reduced to 0.5 cm?

6. $\lambda = 589nm$. $d = 1cm$. $T = 89\% \rightarrow \frac{d}{2}$

$$I = I_0 \exp\left(-\frac{2\omega k}{c} z\right) \quad T = \frac{I(d=1m)}{I_0} = \exp\left(-\frac{2\omega k}{c}\cdot d\right) = 0.89$$

$$T\left(\frac{d}{2}\right) = \frac{I\left(\frac{d}{2}\right)}{I_0} = \exp\left(-\frac{2\omega k}{c}\cdot\frac{d}{2}\right) = (0.89)^{1/2} = 0.943 = 94.3\%$$

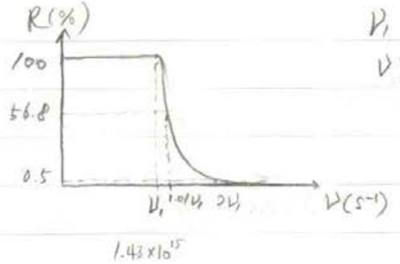
4. Calculate the reflectivity of sodium in the frequency ranges ν > ν_1 and ν < ν_1 using the theory for free electrons without damping. Sketch R versus frequency.

7. Free electrons without damping

$$\hat{n}^2 = 1 - \frac{e^2 N_f}{4\pi^2 \varepsilon_0 m \nu^2} \rightarrow \nu_1^2 = \frac{e^2 N_f}{4\pi^2 \varepsilon_0 m} \quad (N_f : \text{free electrons}/cm^3)$$

$\nu > \nu_1$: $\hat{n}^2 > 0 \rightarrow \hat{n}$: real $\rightarrow k = 0$ $\quad \therefore R = \frac{(n-1)^2}{(n+1)^2}$

$\nu < \nu_1$: $\hat{n}^2 < 0 \rightarrow \hat{n}$: pure imaginary $\rightarrow n = 0$ $\quad \therefore R = \frac{1+k^2}{1-k^2} = 1$



$\nu_1 = 14.3 \times 10^{14}/s$ for Na

$\nu > \nu_1$ 일 때, $\hat{n}^2 = n^2 = 1 - \frac{\nu_1^2}{\nu^2}$, $n = \sqrt{1 - \frac{1}{(\nu/\nu_1)^2}}$

ν/ν_1	n	R
1	0	1
1.01	0.140	0.568
1.1	0.417	0.170
1.5	0.745	0.021
2	0.866	0.005

ν이 증가함에 따라 R은 급격히 0으로 수렴한다.

5. Calculate the reflectivity of gold at ν = 9 x 10^{12} s^{-1} from its conductivity. Is the reflectivity increasing or decreasing at this frequency when the temperature is increased? Explain.

metal, small frequency $< 10^{13} (s^{-1}) \rightarrow$ Hagen - Rubens relation 적용가능

$$R = 1 - 4\sqrt{\frac{\nu}{\sigma_0}\pi\varepsilon_0} \qquad \text{gold} : \rho_0 = 2.35\ \mu\Omega\cdot cm$$

$$= 2.35 \times 10^{-8}\ \Omega\cdot m \rightarrow \sigma_0 = 4.255 \times 10^7\ (\Omega^{-1}\cdot m^{-1})$$

$$\therefore R = 1 - 4\sqrt{\frac{9\times10^{12}}{4.255\times10^7}\pi\cdot 8.854\times10^{-12}} = 0.9903 \quad \therefore R = 99.03\%$$

pure metal 에서 온도가 증가하면 일반적으로 σ_0 이 작아진다. 따라서 온도가 증가함에 따라 reflectivity 는 감소하게 된다.

6. Derive the Drude equations from (11.45) and (11.46) by setting $\nu_0 \to 0$.

10. 11.45. $\varepsilon_1 = 1 + \dfrac{e^2 m N_a(\nu_0^2 - \nu^2)}{\varepsilon_0[4\pi^2 m^2(\nu_0^2-\nu^2)^2 + \gamma'^2\nu^2]}$ 11.46. $\varepsilon_2 = \dfrac{e^2 N_a \gamma' \nu}{2\pi\varepsilon_0[4\pi^2 m^2(\nu_0^2-\nu^2)^2+\gamma'^2\nu^2]}$

$\nu_0 \to 0$ $\quad \varepsilon_1 = 1 + \dfrac{e^2 m N_a(-\cancel{\nu^2})}{\varepsilon_0[4\pi^2m^2(+\nu^2)^{\cancel{2}} + \gamma'^2\cancel{\nu^2}]} = 1 - \dfrac{e^2 \cancel{m} N_a / 4\pi^2\varepsilon_0 m^{\cancel{2}}}{\nu^2 + \gamma'^2/4\pi^2m^2}$ $(N_a = N_f)$ one osillator per atom

$\downarrow$

$\gamma' = \gamma = \dfrac{N_f e^2}{\sigma_0}$ $\quad = 1 - \dfrac{\nu_1^2}{\nu^2 + \gamma'^2/4\pi^2m^2}$, $\left(\dfrac{\gamma'}{2\pi m} = \dfrac{N_f e^2}{2\pi\sigma_0 m} = \dfrac{\nu_1^2 \cdot 2\pi\varepsilon_0}{\sigma_0}\right.$

$\nu_1 = \sqrt{\dfrac{e^2 N_f}{4\pi^2\varepsilon_0 m}}$ $\quad = 1 - \dfrac{\nu_1^2}{\nu^2+\nu_2^2}$ $\quad = \nu_2$

$\nu_2 = \dfrac{2\pi\varepsilon_0\nu_1^2}{\sigma_0}$

$$\varepsilon_2 = \frac{e^2 N_a \gamma' \nu}{2\pi\varepsilon_0[4\pi^2m^2\cdot(\nu^2)^2 + \gamma'^2\nu^2]} = \frac{\gamma'}{\nu}\cdot\frac{1}{2\pi\varepsilon_0}\cdot\frac{e^2N_a}{(4\pi^2m^2\nu^2+\gamma'^2)} = \frac{\gamma'}{\nu}\cdot\frac{1}{2\pi\cancel{\varepsilon_0}m}\cdot\frac{e^2N_a/4\pi^2\varepsilon_0 m}{\cancel{m}\nu^2/\cancel{\varepsilon_0} + \gamma'^2/4\pi^2\cancel{\varepsilon_0}m^2}$$

$$= \frac{\gamma'}{2\pi m\nu}\cdot\frac{\nu_1^2}{\nu^2+\gamma'^2/4\pi^2m^2} = \frac{\nu_2}{\nu}\cdot\frac{\nu_1^2}{\nu^2+\nu_2^2}$$