2014-2 항공기 구조역학 Homework #3

Problem 7.1. Torsion of a bimetallic bar
A circular bar is constructed by bonding an aluminum shell around a solid steel cylinder. The radius of the steel cylinder is $R_S = 10$ mm, and the outer radius of the aluminum shell is $R_A = 20$ mm. The overall length of the bar is given by $L = 1$ m, and a torque $T = 1$ kN·m is applied at the ends. The shear moduli for the aluminum and steel are $G_A = 28$ GPa and $G_S = 76$ GPa, respectively. *(1)* Find the maximum shear stress in the steel and in the aluminum. *(2)* Determine the total twist angle of the bar. *(3)* Determine the torsional stiffness. *(4)* Find the allowable torque for a safety factor of 2 when the yield stresses for both materials is 300 MPa.

Problem 7.2. Torsion of a circular bar with hollow segment
The cylindrical bar shown in fig. 7.9 consists of two segments; the left segment is clamped at point **R**, $\Phi_1(0) = 0$. The left segment of length L is a solid circular bar of radius R_O, while the right segment of length L is a hollow circular bar of inner radius R_i. A moment Q_1 is applied at point **T**. *(1)* Determine the twist angle at point **T**. *(2)* Determine the equivalent torsional stiffness, H, for the complete bar, defined as $H = \Phi_1(2L)/Q_1$. *(3)* Determine ratio of maximum shear stress in the two sections.

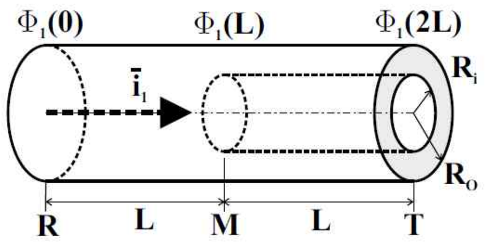

Fig. 7.9. Circular bar with hollow segment.

Problem 7.3. Torsion of a circular bar with hollow segment
The cylindrical bar shown in fig. 7.9 consists of two segments, clamped at point **R** and **T**, $\Phi_1(0) = \Phi_1(2L) = 0$. The left segment of length L is a solid circular bar of radius R_O, while the right segment of length L is a hollow circular bar of inner radius R_i. A moment Q_1 is applied at point **M**. *(1)* Determine the torque carried in each segment. *(2)* Determine the twist angle at point **M**. *(3)* Determine the equivalent torsional stiffness, H, at point **M**, defined as $H = \Phi_1(L)/Q_1$. *(4)* Determine the maximum shear stress in each segment.

Problem 7.4. Torsion of a hollow bar

A circular bar of radius $R = 200$ mm is replaced by a hollow bar of inner and outer radii R_i and R_o, respectively, with $R_0/R_i = 2$. If the two bars are made of the same material and can carry the same maximum torque, determine *(1)* the outer radius of the hollow bar, R_o, and *(2)* the mass ratio for the hollow and solid bars.

Problem 7.6. Pressure vessel subjected to combined loading

The experimental set-up depicted in fig. 7.13 is aimed at studying the behavior of materials under complex stress states. A thin-walled pressure vessel of radius $R = 11$ mm and thickness $t = 2.0$ mm is subjected to an internal pressure p_i. At the same time, a normal force, N, and a torque, Q, are applied to the sample. In a specific experiment, the applied normal force is $N = 16$ kN and the internal pressure $p_i = 20$ MPa. The applied torque is slowly increased. The first permanent deformations are observed at the outer surface of the sample when $Q = 120$ N·m. *(1)* Find the yield stress for the material if it is assumed to follow von Mises' yield criterion. *(2)* Find the yield stress for the material if it is assumed to follow Tresca's yield criterion. *(3)* Find and plot the yield surface in the space defined by the three loading components, the internal pressure, the applied axial force, and the applied torque.

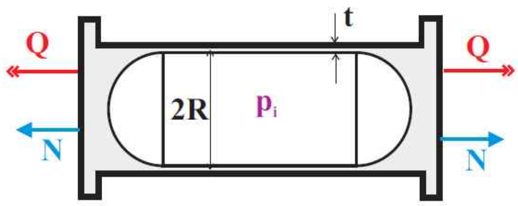

Fig. 7.13. Pressure vessel subjected to internal pressure, external torque and axial force.

Problem 7.7. Beam with circular section under bending and torsion

Consider a cantilevered beam of length $L = 1$ m with a circular cross-section of inner radius $R_i = 45$ mm and outer radius $R_o = 50$ mm. The beam is subjected to a tip torque $Q = 7$ kN·m and a tip transverse load P. Find the maximum allowable transverse load $P_{\max}$ if the allowable stress for the material is $\sigma_{\text{allow}} = 450$ MPa. Note: for a hollow circular section, $H_{22}^c = H_{33}^c = \pi E(R_o^4 - R_i^4)/4$.