1. 교재 p. 28, Prob. 2-11.

$$P(f_X, f_Y) = G(f_X, f_Y)\, XY \mathrm{comb}(X f_X)\, \mathrm{comb}(Y f_Y),$$

where we have used the similarity theorem and the fact that the Fourier transform of a comb function is another comb function. Further simplification results from the following relation:

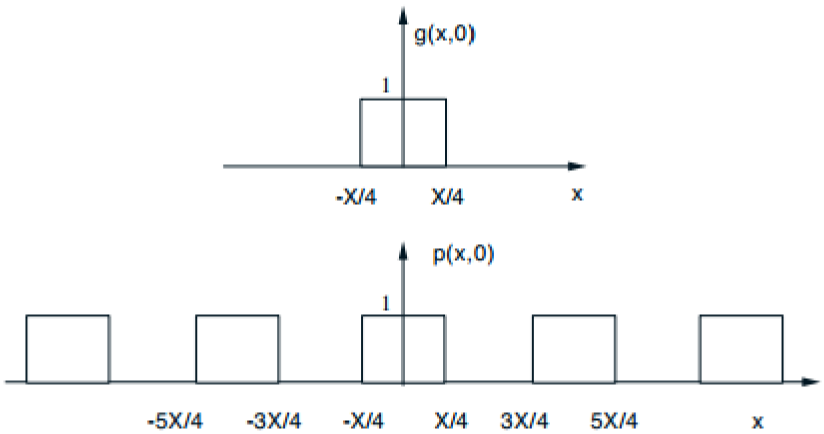
$$\begin{aligned} XY \mathrm{comb}(X f_X)\, \mathrm{comb}(Y f_Y) &= XY \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(X f_X - n, Y f_Y - m) \\ &= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(f_X - \frac{n}{X}, f_Y - \frac{m}{Y}), \end{aligned}$$

where we have used the fact that $\delta(ax, by) = \frac{1}{|a,b|}\delta(x, y)$. We have assumed in the above that $X \geq 0, Y \geq 0$.

(b) The Fourier transform of the given $g(x, y)$ is found as follows:

$$\mathcal{F}\{g(x, y)\} = \frac{XY}{4} \mathrm{sinc}\left(\frac{X}{2} f_X\right) \mathrm{sinc}\left(\frac{Y}{2} f_Y\right),$$

where the similarity theorem has been used. The figure below shows sketches of $g(x, 0)$ and $p(x, 0)$ in this case.



2. 교재 p. 29, Prob. 2-13.

2-13. The object $U_o(x, y)$ has a band-unlimited spectrum, while the transfer function $H(f_X, f_Y)$ of the system is bandlimited to the region $|f_X| \leq B_X, |f_Y| \leq B_Y$. Because of the bandlimitation on H, it is possible to write

$$H(f_X, f_Y) = H(f_X, f_Y)\,\mathrm{rect}\left(\frac{f_X}{2B_X}\right)\mathrm{rect}\left(\frac{f_Y}{2B_Y}\right).$$

Since the imaging system is both linear and invariant, the image and object spectra, G_i and G_o, respectively, can be related by

$$G_i(f_Y, f_Y) = H(f_X, f_Y)\,G_o(f_X, f_Y) = H(f_X, f_Y)\left[\mathrm{rect}\left(\frac{f_X}{2B_X}\right)\mathrm{rect}\left(\frac{f_Y}{2B_Y}\right)G_o(f_X, f_Y)\right].$$

From this equation we can see directly that the output spectrum can be viewed as resulting from the application of a new fictitious object with spectrum

$$G_o'(f_X, f_Y) = \mathrm{rect}\left(\frac{f_X}{2B_X}\right)\mathrm{rect}\left(\frac{f_Y}{2B_Y}\right)G_o(f_X, f_Y).$$

In the space domain, the relation between the fictitious object and the actual object is

$$\begin{aligned} U_o'(x, y) &= U_o(x, y) \otimes 4B_X B_Y \mathrm{sinc}(2B_X x)\mathrm{sinc}(2B_Y y) \\ &= 4B_X B_Y \iint_{-\infty}^{\infty} U_o(\xi, \eta)\,\mathrm{sinc}\left[2B_X(x-\xi)\right]\mathrm{sinc}\left[2B_Y(y-\eta)\right] d\xi d\eta. \end{aligned}$$

Since U_o' is bandlimited, it can be reconstructed from samples taken at the Nyquist rate, i.e. samples taken at coordinates $x_n = \frac{n}{2B_X}, y_m = \frac{m}{2B_Y}$. The sampled object which will yield U_o' after low pass filtering is given by

$$\begin{aligned} \hat{U}_o'(x, y) &= \mathrm{comb}(2B_X x)\,\mathrm{comb}(2B_Y y)\,U_o'(x, y) \\ &= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} U_o'\left(\frac{n}{2B_X x}, \frac{m}{2B_Y y}\right)\delta\left(x - \frac{n}{2B_X}, y - \frac{m}{2B_Y}\right). \end{aligned}$$

Substituting the expression derived above for U_o',

$$\begin{aligned} \hat{U}_o'(x, y) &= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left[\iint_{-\infty}^{\infty} U_o(x, y)\,\mathrm{sinc}(n - 2B_X\xi)\,\mathrm{sinc}(m - 2B_Y\eta)\,d\xi d\eta\right] \\ &\times\ \delta\left(x - \frac{n}{2B_X}, m - \frac{m}{2B_Y}\right). \end{aligned}$$

This array of point sources will yield the same image as the original object $U_o(x, y)$.

3. 교재 p. 61, Prob. 3-5.

3-5. Using Eq. (3-63) we have the following:

(a) For a circular aperture of diameter d:

$$A\left(\frac{\alpha}{\lambda},\frac{\beta}{\lambda};0\right)=\mathcal{B}\left\{\text{circ}\left(\frac{2r}{d}\right)\right\}\Big|_{\substack{f_X=\alpha/\lambda\\ f_Y=\beta/\lambda}}.$$

Using the similarity theorem for Fourier-Bessel transforms (Eq. (2-34)) and the Fourier-Bessel transform pair of Eq. (2-35),

$$A\left(\frac{\alpha}{\lambda},\frac{\beta}{\lambda};0\right)=\frac{d^2}{4}\frac{J_1\left(\frac{2\pi\rho d}{2}\right)}{\frac{d\rho}{2}}=\frac{d}{2}\frac{J_1(\pi\rho d)}{\rho}.$$

Finally, note that $\rho=\sqrt{f_X^2+f_Y^2}=\sqrt{\left(\frac{\alpha}{\lambda}\right)^2+\left(\frac{\beta}{\lambda}\right)^2}$ yielding

$$A\left(\frac{\alpha}{\lambda},\frac{\beta}{\lambda};0\right)=\frac{d^2}{4}\frac{J_1\left(\frac{2\pi\rho d}{2}\right)}{\frac{d\rho}{2}}=\frac{d}{2}\frac{J_1\left(\pi\sqrt{\left(\frac{\alpha}{\lambda}\right)^2+\left(\frac{\beta}{\lambda}\right)^2}\,d\right)}{\sqrt{\left(\frac{\alpha}{\lambda}\right)^2+\left(\frac{\beta}{\lambda}\right)^2}}.$$

(b) A circular opaque disk of diameter d can be modeled by the following amplitude transmittance function:

$$t_A(x,y)=1-\text{circ}\left(\frac{2r}{d}\right).$$

From the linearity theorem of Fourier analysis it follows that the angular spectrum of this structure is

$$A\left(\frac{\alpha}{\lambda},\frac{\beta}{\lambda};0\right)=\delta\left(\frac{\alpha}{\lambda},\frac{\beta}{\lambda}\right)-\frac{d}{2}\frac{J_1\left(\pi\sqrt{\left(\frac{\alpha}{\lambda}\right)^2+\left(\frac{\beta}{\lambda}\right)^2}\,d\right)}{\sqrt{\left(\frac{\alpha}{\lambda}\right)^2+\left(\frac{\beta}{\lambda}\right)^2}}.$$