1. 교재 p. 94, Prob. 4-12.

(a) The Fourier transform of the amplitude transmittance function is

$$\mathcal{F}\{t_A(\xi)\} = \sum_{n=-\infty}^{\infty} c_n \mathcal{F}\{e^{j\frac{2\pi n\xi}{L}}\} = \sum_{n=-\infty}^{\infty} c_n \delta\left(f_X - \frac{n}{L}\right).$$

Assuming unit amplitude, normally incident plane wave illumination, the intensity in any order will be proportional to the squared magnitude of the Fourier coefficient associated with that order. More generally, for arbitrary strength of illumination, the diffraction efficiency of a given order is the squared magnitude of the Fourier coefficient of the delta function corresponding to that order. Thus

$$\eta_k \;=\; |c_k|^2.$$

(b) We must find the Fourier coefficients of the amplitude transmittance function

$$t_A(\xi) = \left|\cos\left(\frac{\pi\xi}{L}\right)\right|.$$

Do so as follows:

$$\begin{aligned} c_k &= \frac{1}{L}\int_{-L/2}^{L/2} \left|\cos\left(\frac{\pi\xi}{L}\right)\right| e^{-j\frac{2\pi k\xi}{L}}\, d\xi = \frac{1}{L}\mathcal{F}\left\{\text{rect}\left(\frac{\xi}{L}\right)\cos\left(\frac{\pi\xi}{L}\right)\right\}_{f_X=k/L} \\ &= \frac{1}{2L}\left(L\text{sinc}\left[L\left(f_X - \frac{1}{2L}\right)\right] + L\text{sinc}\left[L(f_X + \frac{1}{2L})\right]\right)_{f_X=k/L} \\ &= \frac{1}{2}\left[\text{sinc}\left(\frac{2k-1}{2}\right) + \text{sinc}\left(\frac{2k+1}{2}\right)\right]. \end{aligned}$$

The diffraction efficiency is seen to be

$$\eta_k = |c_k|^2 = \frac{1}{4}\left[\text{sinc}\left(\frac{2k-1}{2}\right) + \text{sinc}\left(\frac{2k+1}{2}\right)\right]^2.$$

For the particular case of the first diffraction order ($k = 1$),

$$|c_1|^2 = \frac{1}{4}\left[\text{sinc}\left(\frac{1}{2}\right) + \text{sinc}\left(\frac{3}{2}\right)\right]^2 = \frac{1}{4}\left[\frac{2}{\pi} - \frac{2}{3\pi}\right]^2 = \frac{4}{9\pi^2} = 4.5\%.$$

2. 교재 p. 95, Prob. 4-14.

We begin by writing an equation for the amplitude transmittance of the grating:

$$\begin{aligned} t_A(x) &= 1 - \left[(1-e^{j\phi}) \times (\text{square wave})\right] \\ &= 1 - \left[(1-e^{j\phi}) \times \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n x}{L}}\right] \end{aligned}$$

where,

$$c_n = \frac{1}{L}\int_{-\infty}^{\infty} \text{rect}\left(\frac{\xi}{L/2}\right) e^{-j\frac{2\pi n \xi}{L}}\, d\xi = \frac{1}{L}\mathcal{F}\left\{\text{rect}\left(\frac{x}{L/2}\right)\right\}_{f_X = n/L} = \frac{1}{2}\text{sinc}\left(\frac{n}{2}\right).$$

Continuing,

$$\begin{aligned} \mathcal{F}\{t_A(x)\} &= \delta(f_X) - (1-e^{j\phi}) \sum_{n=-\infty}^{\infty} \frac{1}{2}\text{sinc}\left(\frac{n}{2}\right) \mathcal{F}\left\{e^{j\frac{2\pi n x}{L}}\right\} \\ &= \delta(f_x) - (1-e^{j\phi}) \sum_{n=-\infty}^{\infty} \frac{1}{2}\text{sinc}\left(\frac{n}{2}\right) \delta\left(f_X - \frac{n}{L}\right). \end{aligned}$$

(a) Now finding the diffraction efficiency of the first order,

$$\begin{aligned} \eta_1 &= \eta_{-1} = \left|1-e^{j\phi}\right|^2 \left[\frac{1}{2}\text{sinc}\left(\frac{1}{2}\right)\right]^2 = \frac{1}{4}\left(\frac{2}{\pi}\right)^2 (2 - 2\cos\phi) \\ &= \frac{2}{\pi^2}(1-\cos\phi). \end{aligned}$$

(b) To maximize η_1 and η_{-1}, we require $\cos\phi = -1$, or $\phi = \pi$. In this case the diffraction efficiency becomes

$$\eta_1 = \eta_{-1} = \frac{4}{\pi^2} = 40.5\%.$$

3. 교재 p. 125, Prob. 5-11.

Fourier planes will be found at the following locations:

- In the plane where the illumination beam comes to focus; i.e. distance f to the right of the object.
- In the plane where the above Fourier plane is imaged by the lens. According to the lens law, this will be at distance $2f$ to the right of the lens.

There will be only one image plane, namely the plane where the lens law is satisfied for an object $3f$ in front of the lens. We have

$$\frac{1}{z_i} + \frac{1}{3f} = \frac{1}{f}$$

from which it follows that

$$z_i = \frac{3f}{2}$$

to the right of the lens.