## SEOUL NATIONAL UNIVERSITY

## SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

1. A radar tracks the flight of a projectile (see Figure 1). At time $t$, the radar measures the horizontal component $v_{x}(t)$ and the vertical component $v_{y}(t)$ of the projectile's velocity and its range $R(t)$ and elevation $\pi(t)$. Are these measurements sufficient to compute the horizontal distance $D$ from the radar to the launch point of the projectile? If so, derive the expression for $D$ as a function of $g$ and the measured values $v_{x}(t), v_{y}(t), R(t)$, and $\pi(t)$.


Figure 1
2. The motor in Figure $\mathbf{2}$ lifts the mass $m_{L}$ by winding up the cable with a force $F_{A}$. The center of pulley $B$ is fixed. The pulley inertias are $I_{B}$ and $I_{C}$. Pulley $C$ has mass $m_{c}$. Derive the equation of motion in terms of the speed $v_{A}$ of point $A$ on the cable with the force $F_{A}$ as input.


Figure 2
3. The geared system shown in Figure 3 represents an elevator system. The motor has an inertia $I_{1}$ and supplies a torque $T_{1}$. Neglect the inertias of the gears, and assume that the cable does not slip on the pulley. Derive an expression for the equivalent inertia $I_{e}$ felt on the input shaft (shaft 1). Then derive the dynamic model of the system in terms of the speed $\omega_{1}$ and the applied torque $T_{1}$. The pulley radius is $R$.


Figure 3
4. The lead screw (also called a power screw or a jack screw) is used to convert the rotation of a motor shaft into a translational motion of mass $\boldsymbol{m}$ (see Figure 4). For one revolution of the screw, the mass translates a distance $L$ (called the screw lead). As felt on the mirror shaft, the translating mass appears as an equivalent inertia. Use kinematic energy equivalence to derive an expression for the equivalent inertia. Let $I_{s}$ be the inertia of the screw.


Figure 4
5. At time $t=0$, the operator of the road roller disengages the transmission so that the vehicle rolls down the incline (see Figure 5). Derive a dynamic model for the vehicle's speed. The two rear wheels weigh 250 kg each and have a radius of 1 m . The front wheel weighs 400 kg and has a radius of 0.5 m . The vehicle body weights 4500 kg . Assume the wheels roll without slipping.


Figure 5
6. A conveyor drive system to produce translation of the load is shown in Figure 6-(a). The redactor is a gear pair that reduces the motor speed by a factor of $10: 1$. The motor inertia is $I_{I}=0.002 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. The redactor inertia as felt on the motor shaft is $I_{2}=0.003 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. Neglect the inertia of the tachometer, which is used to measure the speed for control purposes. The properties of the remaining elements are given here.

## Sprockets:

Sprocket 1: radius $=0.05 \mathrm{~m}$ weight $=9 \mathrm{~N}$
Sprocket 2: radius $=0.15 \mathrm{~m}$ weight $=89 \mathrm{~N}$
Chain weight: 107 N
Drive shaft: radius $=0.04 \mathrm{~m}$ weight $=22 \mathrm{~N}$
Drive wheels (four of them): radius $=0.2 \mathrm{~m}$ weight $=89 \mathrm{~N}$
Drive chains (two of them): weight $=670 \mathrm{~N}$ each
Load friction torque measured at the drive shaft: $54 \mathrm{~N} \cdot \mathrm{~m}$
Load weight: $\mathbf{4 5 0} \mathbf{N}$
a. Derive the equation of motion of the conveyor in terms of the motor velocity $\omega_{1}$, with the motor torque $T_{1}$ as input.
b. Suppose the motor torque is constant at $10 \mathrm{~N} \cdot \mathrm{~m}$. Determine the resulting motor velocity $\omega_{1}$ and load velocity v as functions of time, assuming the system starts from rest.
c. The profile of a desired velocity for the load is shown in Figure 6 -(b), where $v_{0}=1 \mathbf{~ m} / \mathrm{s}$, $t_{1}=t_{3}=0.5 \mathrm{~s}$, and $t_{2}=2 \mathrm{~s}$. Use the equation of motion found in part (a) to compute the required motor torque for each part of the profile.


Figure 6-(a)


Figure 6-(b)
7. The vehicle operates with shaft torque $T_{s}$ and brake torque $\boldsymbol{T}_{b}$. Derive a dynamic model for vehicle. (Use some assumptions and coefficients if it needed)
Input: $u_{1}=T_{s}, u_{2}=T_{b}$,
State: ${ }^{x=\text { distance }}$
$\dot{x}=$ velocity


Figure 7

