1장 1.a.

풀

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)} \qquad y(0) = -1$$

$$2(y-1)\,dy = 3x^2+4x+2\,dx$$

$$(2y-2)\,dy = (3x^2+4x+2)\,dx$$

적분

$$y^2-2y = x^3+2x^2+2x+C$$

$$y^2-2y+1 = x^3+2x^2+2x+C+1$$

$$(y-1)^2 = x^3+2x^2+2x+C+1$$

$$y = 1+\sqrt{x^3+2x^2+2x+C+1} \quad \cdots \text{ case } 1$$

or

$$1-\sqrt{x^3+2x^2+2x+C+1} \quad \cdots \text{ case } 2$$

$$y(0) = -1$$

case 1

$$-1 = 1+\sqrt{C+1} \qquad \therefore \text{ 불가.}$$

case 2

$$-1 = 1-\sqrt{C+1} \qquad \therefore\ C = 3$$

$$\therefore\ y = 1-\sqrt{x^3+2x^2+2x+4}$$

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b. $$\frac{dy}{dx} = k(a-y)(b-y)$$

i) $a = b$

$$\frac{dy}{dx} = k(a-y)^2$$

$$\frac{1}{(a-y)^2}\,dy = k\,dx$$

적분

$$\frac{1}{a-y} = kx + C$$

$$a-y = \frac{1}{kx+c}$$

$$a - \frac{1}{kx+c} = y$$

$$y = \frac{akx+ac-1}{kx+c}, \quad \text{and} \quad y(0) = 0$$

$$0 = \frac{ac-1}{c} \quad \therefore c = \frac{1}{a}$$

$$y(x) = \frac{akx+1-1}{kx+\frac{1}{a}} = \frac{a^2kx}{akx+1} \quad \text{when } a=b$$

ii) $a \neq b$

$$\frac{dy}{dx} = k(a-y)(b-y)$$

$$\frac{1}{(a-y)(b-y)}\,dy = k\,dx$$

$$\left(\frac{1}{(b-a)(a-y)} + \frac{1}{(a-b)(b-y)}\right)dy = k\,dx$$

$$\frac{-1}{b-a}\ln(a-y) + \frac{-1}{a-b}\ln(b-y) = kx+c$$

$$-\ln(a-y) + \ln(b-y) = (b-a)(kx+c)$$

$$\ln \frac{b-y}{a-y} = (b-a)(kx+c)$$

$$\frac{b-y}{a-y} = e^{(b-a)(kx+c)}$$

$$b-y = a e^{(b-a)(kx+c)} - y e^{(b-a)(kx+c)}$$

$$\left(\begin{array}{l} y(0)=0 \quad \text{대입} \\ b = a e^{(b-a)c} \Rightarrow e^{(b-a)c} = \frac{b}{a} \end{array} \right)$$

$$b-y = a e^{(b-a)kx} e^{(b-a)c} - y e^{(b-a)kx} e^{(b-a)c}$$

$$= b e^{(b-a)kx} - \frac{b}{a} y e^{(b-a)kx}$$

$$b - b e^{(b-a)kx} = y - \frac{b}{a} y e^{(b-a)kx}$$

$$b(1-e^{(b-a)kx}) = \frac{1}{a} y (a - b e^{(b-a)kx})$$

$$y = \frac{ab(1-e^{(b-a)kx})}{a - b e^{(b-a)kx}}$$

$$\therefore \quad y = \begin{cases} \dfrac{a^2 kx}{akx+1} & ; a = b \\ \dfrac{ab(1-e^{(b-a)kx})}{a-be^{(b-a)kx}} & ; a \neq b \end{cases}$$

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2. a $2xy^3 + 3x^2y^2 \frac{dy}{dx} = 0$, $y(1) = 1$

$$\frac{\partial}{\partial y}(2xy^3) = 6xy^2 , \quad \frac{\partial}{\partial x}(3x^2y^2) = 6xy^2$$

$\therefore$ exact

$$\frac{\partial \psi}{\partial x} = 2xy^3 \Rightarrow \psi = x^2y^3 + f(y)$$

$$\frac{\partial \psi}{\partial y} = 3x^2y^2 \Rightarrow \psi = x^2y^3 + f(x)$$

$$\therefore \psi = x^2y^3 = C$$

$y(1) = 1$ 대입

$\therefore C = 1$

$x^2y^3 = 1$

$$\therefore y = x^{-\frac{2}{3}}$$

b. $3xy + y^2 + (x^2+xy)\frac{dy}{dx} = 0$, $y(2) = 1$

$$\frac{\partial}{\partial y}(3xy+y^2) = 3x+2y$$

$$\frac{\partial}{\partial x}(x^2+xy) = 2x+y$$

$\therefore$ not exact

$$\mu(x) = \exp\left(\int \frac{3x+2y-2x-y}{x^2+xy}\,dx\right)$$

$$= \exp\left(\int \frac{x+y}{x(x+y)}\,dx\right)$$

$$= \exp\left(\int \frac{1}{x}\,dx\right) = \exp(\ln x) = x.$$

∴ 준식에 $\mu(x)$ 곱하면

$$3x^2y + xy^2 + (x^3 + x^2y)\frac{dy}{dx} = 0$$

$$\frac{\partial \psi}{\partial x} = 3x^2y + xy^2 \quad \Rightarrow \psi = x^3y + \frac{1}{2}x^2y^2 + f(y)$$

$$\frac{\partial \psi}{\partial y} = x^3 + x^2y \quad \Rightarrow \psi = x^3y + \frac{1}{2}x^2y^2 + f(x)$$

$$\therefore \psi = x^3y + \frac{1}{2}x^2y^2 = C$$

$y(2) = 1$ 대입

$$8 + \frac{1}{2}4 = C$$

$$\therefore C = 10$$

$$\therefore x^3y + \frac{1}{2}x^2y^2 = 10$$

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3. $$\frac{dy}{dx} = \frac{x+y+1}{x-y+3}$$

Let's $x = X + h$, $y = Y + k$.

$$\frac{x+y+1}{x-y+3} = \frac{X+h+Y+k+1}{X+h-Y-k+3} = \frac{X+Y}{X-Y}$$

$$\begin{aligned} h+k &= -1 \\ h-k &= -3 \end{aligned} \quad \Rightarrow \quad \begin{aligned} h &= -2 \\ k &= +1 \end{aligned}$$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

Let's $u = \frac{Y}{X}$

$$\frac{dY}{dX} = \frac{1 + \frac{Y}{X}}{1 - \frac{Y}{X}}$$

$$uX = Y$$

$$\frac{dY}{dX} = u + X\frac{du}{dX}$$

$$u + X\frac{du}{dX} = \frac{1+u}{1-u}$$

$$+X\frac{du}{dX} = \frac{1+u}{1-u} - u = \frac{1+u-u+u^2}{1-u} = \frac{1+u^2}{1-u}$$

$$\frac{1-u}{1+u^2}\,du = \frac{1}{X}\,dX$$

$$\left(\frac{1}{1+u^2} - \frac{u}{1+u^2}\right)du = \frac{1}{X}\,dX$$

$$\arctan u - \frac{1}{2}\ln(1+u^2) = \ln X + C$$

$u = \frac{Y}{X}$ 대입

$$\arctan\left(\frac{Y}{X}\right) - \frac{1}{2}\ln\left(1+\left(\frac{Y}{X}\right)^2\right) = \ln X + C$$

$X = x+2$, $Y = y-1$ 대입

$$\arctan\left(\frac{y-1}{x+2}\right) - \frac{1}{2}\ln\left(1+\left(\frac{y-1}{x+2}\right)^2\right) = \ln(x+2) + C$$ //

4.

a. $$\frac{dP}{dt} = 0.003P - 0.001P^2 - 0.002$$

b. $$\frac{dP}{dt} = 0.003P - 0.001P^2 - 0.002 = \frac{-1}{1000}(P^2 - 3P + 2)$$

$$\frac{1}{(P-2)(P-1)}\,dP = -\frac{1}{1000}\,dt$$

$$\left(\frac{1}{P-2} - \frac{1}{P-1}\right)dP = -\frac{1}{1000}\,dt$$

$$\ln(P-2) - \ln(P-1) = -\frac{1}{1000}t + C$$

$$\ln\frac{P-2}{P-1} = -\frac{1}{1000}t + C$$

$$\frac{P-2}{P-1} = C_1 e^{-\frac{1}{1000}t}$$

$$P(0) = 1000000$$

$$\underline{\frac{999998}{999999} = C_1} \; //$$

$$P - 2 = (P-1)C_1 e^{-\frac{1}{1000}t}$$

$$P - 2 = PC_1 e^{-\frac{1}{1000}t} - C_1 e^{-\frac{1}{1000}t}$$

$$P\left(1 - C_1 e^{-\frac{1}{1000}t}\right) = 2 - C_1 e^{-\frac{1}{1000}t}$$

$$P = \frac{2 - C_1 e^{-\frac{1}{1000}t}}{1 - C_1 e^{-\frac{1}{1000}t}}, \quad C_1 \text{ 대입}$$

$$P = \frac{2 - \frac{999998}{999999}e^{-\frac{1}{1000}t}}{1 - \frac{999998}{999999}e^{-\frac{1}{1000}t}}$$

$$\underline{P = \frac{1999998 - 999998e^{-\frac{1}{1000}t}}{999999 - 999998e^{-\frac{1}{1000}t}}} \; //$$