3.4

$$u_0 = (u_{st})_0 R_d = \frac{p_0}{k}\left(\frac{1}{\sqrt{\left(1-(\omega/\omega_n)^2\right)^2 + (2\zeta\omega/\omega_n)^2}}\right)$$

i . 20rpm으로 진동할경우 $\omega/\omega_n = 0.1$

$\zeta = 0$ 일 경우

$$0.2 = \frac{p_0}{k}\frac{1}{\sqrt{\left(1-0.1^2\right)^2}}, \quad \frac{p_0}{k} = 0.198$$

$\zeta = 0.25$ 일 경우

$$u_0 = 0.198\frac{1}{\sqrt{\left(1-0.1^2\right)^2 + (2\times 0.25\times 0.1)^2}} = 0.1997$$

ii. 180rpm으로 진동할경우 $\omega/\omega_n = 0.9$

$\zeta = 0$ 일 경우

$$1.042 = \frac{p_0}{k}\frac{1}{\sqrt{\left(1-0.9^2\right)^2}}, \quad \frac{p_0}{k} = 0.19798$$

$\zeta = 0.25$ 일 경우

$$u_0 = 0.19798\frac{1}{\sqrt{\left(1-0.9^2\right)^2 + (2\times 0.25\times 0.9)^2}} = 0.4053$$

iii. 600rpm으로 진동할경우 $\omega/\omega_n = 3$

$\zeta = 0$ 일 경우

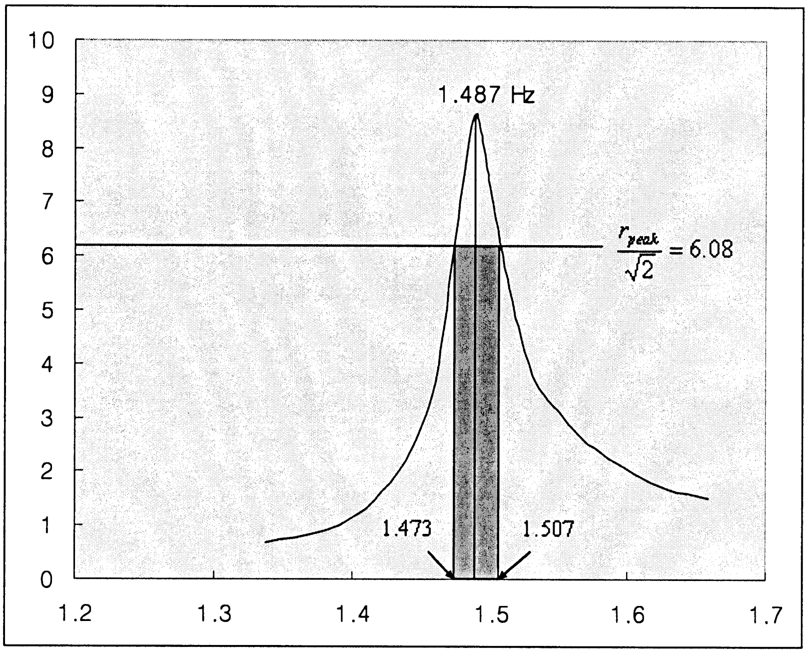
$$0.0248 = \frac{p_0}{k}\frac{1}{\sqrt{\left(1-3^2\right)^2}}, \quad \frac{p_0}{k} = 0.1984$$

$\zeta = 0.25$ 일 경우

$$u_0 = 0.1984\frac{1}{\sqrt{\left(1-3^2\right)^2 + (2\times 0.25\times 3)^2}} = 0.0244$$

=> 고유진동수 근처의 진동수로 진동할 때는 감쇠비가 존재할 때 변위가 줄어드는 것이 확연히 보이지만 그 이외의 진동수로 진동할 때는 별 차이가 없다.

3.11



$\omega_n = 2\pi f_n = 2\pi \cdot 1.487 = 9.34 rad/s$

$\zeta = \frac{f_b - f_a}{2f_n} = \frac{1.507 - 1.473}{2 \times 1.487} = 0.0114$,

3.15

$$TR = \left[\frac{1 + (2\zeta\omega/\omega_n)^2}{\left(1-(\omega/\omega_n)^2\right)^2 + (2\zeta\omega/\omega_n)^2} \right]^{\frac{1}{2}}$$

$\zeta = 0$, $TR = 0.1$ 이므로

$0.1 = \frac{1}{\sqrt{\left(1-(\omega/\omega_n)^2\right)^2}}$, $(\omega/\omega_n)^2 = 11$, $\omega/\omega_n = 3.317$

$\omega = 2\pi \frac{1500}{60} = 157.08\ rad/s$, $\omega_n = \frac{157.08}{3.317} = 47.36\ rad/s$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{2000/386}} = 47.36$, $\Rightarrow k = 11621.60\ lb/in = 11.6\ kips/in$

3.20

$$\frac{u_0}{u_{g0}} = R_a = \left(\frac{\omega}{\omega_n}\right)^2 R_d = \frac{(\omega/\omega_n)^2}{\sqrt{\left(1-(\omega/\omega_n)^2\right)^2 + \left(2\zeta(\omega/\omega_n)\right)^2}}$$

$\omega/\omega_n = \beta$ 이 매우 클 경우 $R_a = 1$ 에 가까워지고 $\frac{u_0}{u_{g0}} = 1$ 일 때 최고의 정확도를 얻을 수 있다.

$$\frac{\beta^2}{\sqrt{\left(1-\beta^2\right)^2 + (2\zeta\beta)^2}} = 1$$

$$\Rightarrow \left(1-\beta^2\right)^2 + (2\zeta\beta)^2 = \beta^4$$

$$\Rightarrow 1 - 2\beta^2 + 4\zeta^2\beta^2 = 0$$

$$\Rightarrow \zeta^2 = \frac{1}{2} - \frac{1}{4\beta^2}, \quad \beta >> 1 \text{ 이므로,}$$

$$\Rightarrow \zeta^2 = 0.5, \quad \zeta = 0.7071$$

3.26

$$p(t) = a_0 + \sum_{j=1}^{\infty} a_j \cos(j\omega_0 t) + \sum_{j=1}^{\infty} b_j \sin(j\omega_0 t), \qquad \omega_0 = \frac{2\pi}{T_0}$$

(a) 주어진 하중함수는 even function으로서 $0 \le t \le \frac{T_0}{2}$ 에서 $p(t) = p_0\left(1 - \frac{2}{T_0}t\right)$ 인 함수이다.

$$a_0 = \frac{1}{T_0}\int_0^{T_0} p(t)dt = \frac{2}{T_0}\int_0^{\frac{T_0}{2}} p(t)dt$$

$$= \frac{2}{T_0}\int_0^{\frac{T_0}{2}} p_0\left(1 - \frac{2}{T_0}t\right)dt$$

$$= \frac{2}{T_0}\left(p_0 t\Big|_0^{\frac{T_0}{2}} - \frac{p_0}{T_0}t^2\Big|_0^{\frac{T_0}{2}}\right) = \frac{p_0}{2}$$

$$a_j = \frac{2}{T_0}\int_0^{T_0} p(t)\cos(j\omega_0 t)dt = \frac{4}{T_0}\int_0^{\frac{T_0}{2}} p(t)\cos\left(\frac{2\pi j}{T_0}t\right)dt$$

$$= \frac{4p_0}{T_0}\int_0^{\frac{T_0}{2}}\left(1-\frac{2}{T_0}t\right)\cos\left(\frac{2\pi j}{T_0}t\right)dt$$

$$= \frac{4p_0}{T_0}\left[\frac{T_0}{2\pi j}\sin\left(\frac{2\pi j}{T_0}t\right)\Bigg|_0^{\frac{T_0}{2}} - \frac{2}{T_0}\int_0^{\frac{T_0}{2}} t\cos\left(\frac{2\pi j}{T_0}t\right)dt\right]$$

$$= -\frac{8p_0}{T_0^{\,2}}\int_0^{\frac{T_0}{2}} t\cos\left(\frac{2\pi j}{T_0}t\right)dt$$

$$= -\frac{8p_0}{T_0^{\,2}}\left[\frac{T_0 t}{2\pi j}\sin\left(\frac{2\pi j}{T_0}t\right)\Bigg|_0^{\frac{T_0}{2}} - \frac{T_0}{2\pi j}\int_0^{\frac{T_0}{2}}\sin\left(\frac{2\pi j}{T_0}t\right)dt\right]$$

$$= -\frac{8p_0}{T_0^{\,2}}\left[\left(\frac{T_0}{2\pi j}\right)^2\cos\left(\frac{2\pi j}{T_0}t\right)\Bigg|_0^{\frac{T_0}{2}}\right]$$

$$= -\frac{2p_0}{\pi^2 j^2}\left[\cos(\pi j)-1\right]$$

$$\therefore a_j = \begin{cases} \dfrac{4p_0}{\pi^2 j^2} & j = 1,3,5,.. \\ 0 & j = 2,4,6,.. \end{cases}$$

$b_j = 0 \quad \because p(t)$ is even function.

$$\therefore p(t) = \frac{p_0}{2} + \frac{4p_0}{\pi^2}\sum_{j=1,3,5..}^{\infty}\frac{1}{j^2}\cos(j\omega_0 t)$$

(b)

$$u(t)=\frac{a_0}{k}+\sum_{j=1}^{\infty}\frac{1}{k}\frac{1}{\left(1-\beta_j^{\,2}\right)^2+\left(2\zeta\beta_j\right)^2}\left\{\left[a_j\left(2\zeta\beta_j\right)+b_j\left(1-\beta_j^{\,2}\right)\right]\sin\left(j\omega_0 t\right)+\left[a_j\left(1-\beta_j^{\,2}\right)-b_j\left(2\zeta\beta_j\right)\right]\cos\left(j\omega_0 t\right)\right\}$$

$\beta_j = j\omega_0 / \omega_n$

무감쇠 시스템에서 $\zeta = 0$ 이므로,

$$u(t)=\frac{a_0}{k}+\frac{1}{k}\sum_{j=1,3,5..}^{\infty} a_j \frac{1}{\left(1-\beta_j^{\,2}\right)}\cos\left(j\omega_0 t\right)$$

$$\frac{u(t)}{\left(u_{st}\right)_0}=\frac{1}{2}+\frac{4}{\pi^2}\sum_{j=1,3,5..}^{\infty}\frac{1}{j^2\left(1-\beta_j^{\,2}\right)}\cos\left(j\omega_0 t\right)$$

$\beta_j = 1$ 이면 해를 결정할 수 없다. 즉, $j\frac{2\pi}{T_0}=\frac{2\pi}{T_n}$ 인 경우로 $T_0 = jT_n = T_n, 3T_n, 5T_n, \ldots$

(c) $T_0 / T_n = 2$ 인 경우, $\beta_j = \frac{j}{2}$ 가 되므로

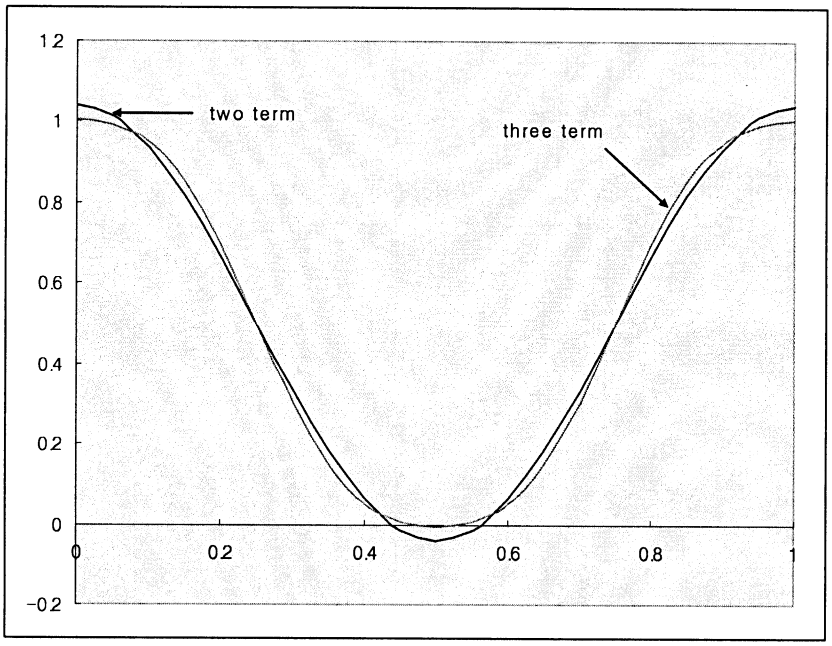
$$\frac{u(t)}{\left(u_{st}\right)_0}=\frac{1}{2}+\frac{4}{\pi^2}\sum_{j=1,3,5,..}^{\infty}\frac{1}{j^2\left(1-j^2/4\right)}\cos\left(\frac{2\pi j}{T_0}t\right)$$

$j=1$ 까지 계산했을 때 (two term)

$$\frac{u(t)}{\left(u_{st}\right)_0}=\frac{1}{2}+\frac{16}{3\pi^2}\cos\left(2\pi\frac{t}{T_0}\right)$$

$j=3$ 까지 계산했을 때 (three term)

$$\frac{u(t)}{\left(u_{st}\right)_0}=\frac{1}{2}+\frac{16}{3\pi^2}\cos\left(2\pi\frac{t}{T_0}\right)-\frac{16}{45\pi^2}\cos\left(6\pi\frac{t}{T_0}\right)$$



위의 결과로 보아서 ~~two term 만 사용~~해도 합리적인 수준으로 수렴하는 것을 볼 수 있다. 그 이유는 급수항의 분모에 j^4이 있기 때문에 j가 커질수록 급수항은 0에 가까워져 무의미한 값을 가지기 때문이다.