4.5

(a) 10

$$u(t)=\frac{1}{m\omega_n}\int_0^t p(\tau)\sin\omega_n(t-\tau)d\tau$$

$$=\frac{1}{m\omega_n}\int_0^t p_0e^{-a\tau}\sin\omega_n(t-\tau)d\tau$$

$$=\frac{p_0}{m\omega_n}\left[-\frac{1}{a}e^{-a\tau}\sin\omega_n(t-\tau)\Big|_0^t-\frac{\omega_n}{a}\int_0^t e^{-a\tau}\cos\omega_n(t-\tau)d\tau\right]$$

$$=\frac{p_0}{m\omega_n a}\sin\omega_n t-\frac{p_0}{ma}\left[-\frac{1}{a}e^{-a\tau}\cos\omega_n(t-\tau)\Big|_0^t+\frac{\omega_n}{a}\int_0^t e^{-a\tau}\sin\omega_n(t-\tau)d\tau\right]$$

$$=\frac{p_0}{m\omega_n a}\sin\omega_n t-\frac{p_0}{ma}\left[-\frac{1}{a}e^{-at}+\frac{1}{a}\cos\omega_n t+\frac{\omega_n}{a}\int_0^t e^{-a\tau}\sin\omega_n(t-\tau)d\tau\right]$$

$$=\frac{p_0}{m\omega_n a}\sin\omega_n t+\frac{p_0}{ma^2}e^{-at}-\frac{p_0}{ma^2}\cos\omega_n t-\frac{p_0\omega_n}{ma^2}\int_0^t e^{-a\tau}\sin\omega_n(t-\tau)d\tau$$

※

$$\frac{p_0}{m\omega_n}\int_0^t e^{-a\tau}\sin\omega_n(t-\tau)d\tau+\frac{p_0\omega_n}{ma^2}\int_0^t e^{-a\tau}\sin\omega_n(t-\tau)d\tau=\frac{p_0}{ma^2}\left[\frac{a}{\omega_n}\sin\omega_n t-\cos\omega_n t+e^{-at}\right]$$

$$\Rightarrow\int_0^t e^{-a\tau}\sin\omega_n(t-\tau)d\tau=\frac{\omega_n}{\left(a^2+\omega_n^{\,2}\right)}\left[\frac{a}{\omega_n}\sin\omega_n t-\cos\omega_n t+e^{-at}\right]$$

이를 대입하면,

$$u(t)=\frac{1}{m\omega_n}\int_0^t p(\tau)\sin\omega_n(t-\tau)d\tau$$

$$=\frac{p_0}{ma^2}\left\{1-\frac{\omega_n^{\,2}}{\left(a^2+\omega_n^{\,2}\right)}\right\}\left[\frac{a}{\omega_n}\sin\omega_n t-\cos\omega_n t+e^{-at}\right]$$

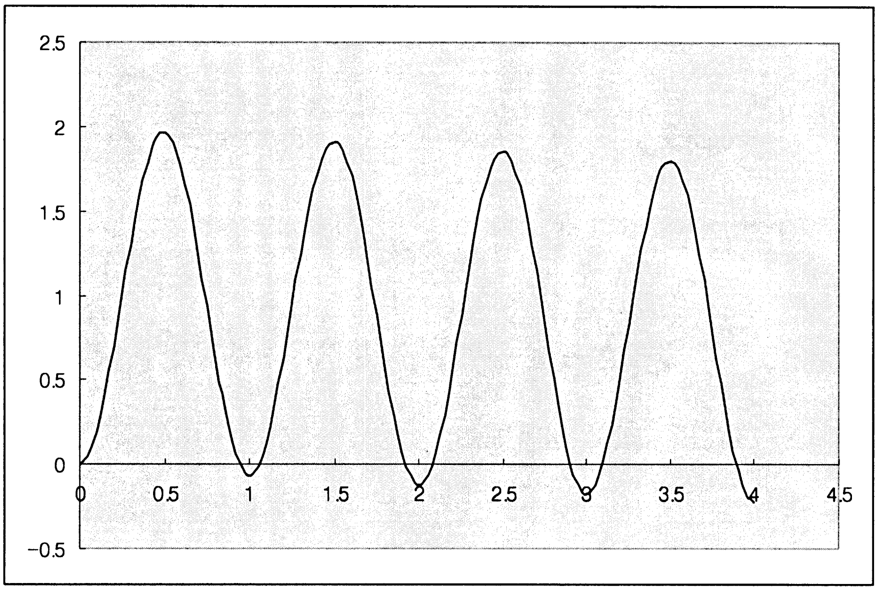
$$=\frac{P_0}{k}\frac{1}{1+(a/\omega_n)^2}\left[\frac{a}{\omega_n}\sin\omega_n t-\cos\omega_n t+e^{-at}\right]$$

$$\therefore\ \frac{u(t)}{(u_{st})_0}=\frac{1}{1+(a/\omega_n)^2}\left[\frac{a}{\omega_n}\sin\omega_n t-\cos\omega_n t+e^{-at}\right]$$

(b) x축을 $t/T_n$, y축을 $u(t)/(u_{st})_0$ 로 하여 도시해보면

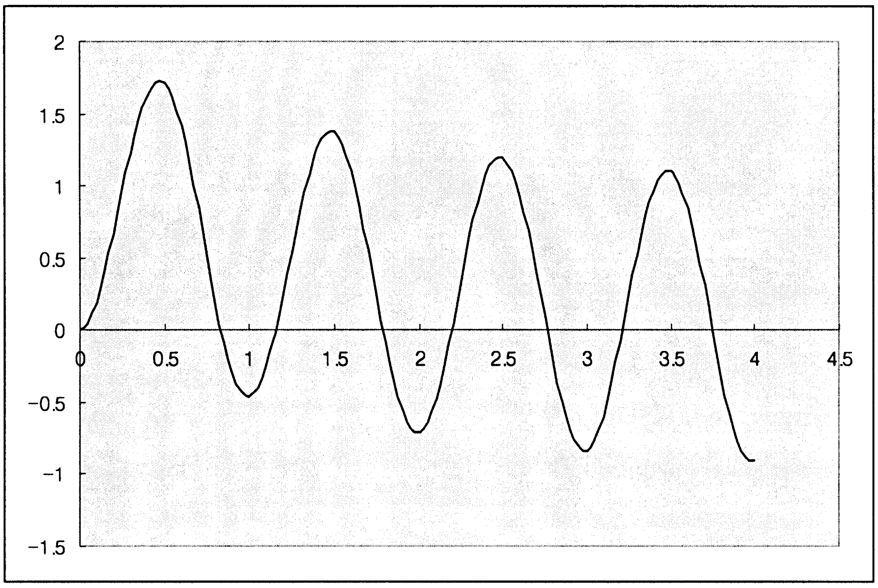
ⅰ. $a/\omega_n = 0.01$ 일때,

$$\frac{u(t)}{(u_{st})_0} = \frac{1}{1+(0.01)^2}\left[0.01\sin\frac{2\pi}{T_n}t - \cos\frac{2\pi}{T_n}t + e^{-\frac{0.02\pi}{T_n}t}\right]$$



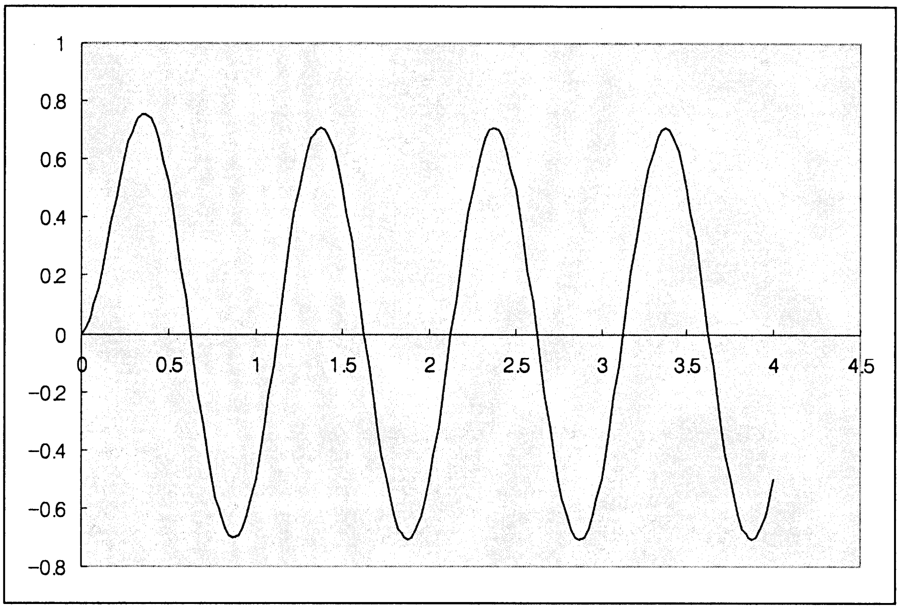

ⅱ. $a/\omega_n = 0.1$ 일때,

$$\frac{u(t)}{(u_{st})_0} = \frac{1}{1+(0.1)^2}\left[0.1\sin\frac{2\pi}{T_n}t - \cos\frac{2\pi}{T_n}t + e^{-\frac{0.2\pi}{T_n}t}\right]$$

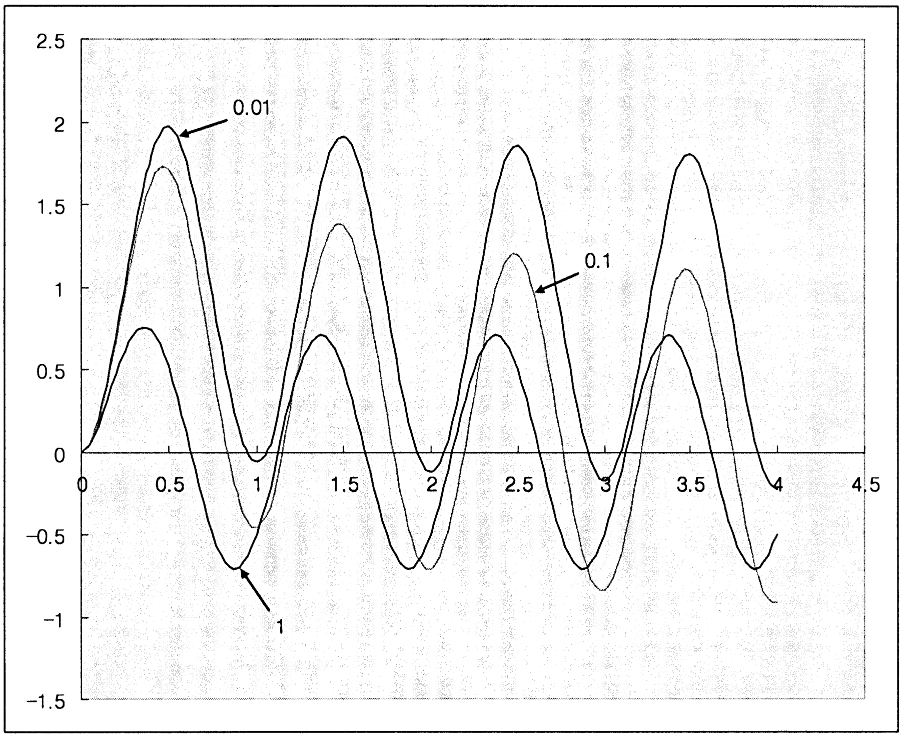


iii. $a/\omega_n = 0.1$ 일때,

$$\frac{u(t)}{(u_{st})_0} = \frac{1}{1+(1)^2}\left[1\sin\frac{2\pi}{T_n}t - \cos\frac{2\pi}{T_n}t + e^{-\frac{2\pi}{T_n}t}\right]$$



<종합>



(c)

$t \to \infty$ 로 가면 $e^{-at} \to 0$ 이 되면서 안정상태에 도달하게 된다.
그때의 진폭은

$$\frac{u(t)}{(u_{st})_0} = \frac{1}{1+(a/\omega_n)^2}\sqrt{\left(\frac{a}{\omega_n}\right)^2+1} = \frac{1}{\sqrt{1+(a/\omega_n)^2}}$$

## 4.17

골조의 횡강성을 구해보면,

$$k_{col} = \frac{12EI}{L^3} = \frac{12\times 30000\times 61.9}{(12\times 12)^3} = 7.463$$

$$k = 2\times 7.463 = 14.926\ kips/in$$

$T_n = 2\pi\sqrt{\frac{m}{k}}$ 에서 $k$ 가 4배가 되었으므로 $T_n$ 은 $1/2$ 배가 된다.

$$\therefore T_n = 0.25\ \sec$$

$$\frac{t_d}{T_n} = \frac{0.2}{0.25} = 0.8 > 0.5,\quad R_d = \frac{u_0}{(u_{st})_0} = 2$$

$$(u_{st})_0 = \frac{p_0}{k} = \frac{4}{14.926} = 0.268\ in$$

$$u_0 = R_d(u_{st})_0 = 2\times 0.268 = 0.536\ in$$

$$M = \frac{6EI}{L^2}u_0 = \frac{6\times 30000\times 61.9}{(12\times 12)^2}0.536 = 288.01\ kip-in$$

$$\sigma = \frac{M}{S} = \frac{288.01}{15.2} = 18.95\ ksi$$

기둥의 밑면이 고정단으로 바뀌면서 최대동적변위 및 최대휨모멘트와 최대휨응력 모두 줄어든 것을 볼 수 있다. 따라서 밑면의 구속정도가 강할수록 구조물은 강성이 강해짐을 알 수 있다.

## 4.21

(a)

i. $p(t) = \frac{p_0}{t_d}t,\quad 0 \le t \le t_d$

$$u(t) = \frac{1}{m\omega_n}\int_0 p(\tau)\sin\omega_n(t-\tau)d\tau$$

$$= \frac{1}{m\omega_n}\frac{p_0}{t_d}\int_0^t \tau \sin\omega_n(t-\tau)d\tau$$

$$= \frac{1}{m\omega_n}\frac{p_0}{t_d}\left[\frac{1}{\omega_n}\cos\omega_n(t-\tau)\tau\Big|_0^t - \frac{1}{\omega_n}\int_0^t \cos\omega_n(t-\tau)d\tau\right]$$

$$= \frac{1}{m\omega_n}\frac{p_0}{t_d}\left[\frac{t}{\omega_n} + \frac{1}{\omega_n^2}\sin\omega_n(t-\tau)\Big|_0^t\right]$$

$$= \frac{1}{m\omega_n}\frac{p_0}{t_d}\left[\frac{t}{\omega_n} - \frac{1}{\omega_n^2}\sin\omega_n t\right]$$

$$= \frac{p_0}{k}\left[\frac{t}{t_d} - \frac{\sin\omega_n t}{\omega_n t_d}\right] = \frac{p_0}{k}\left[\frac{t}{t_d} - \frac{1}{2\pi}\left(\frac{T_n}{t_d}\right)\sin\frac{2\pi t}{T_n}\right]$$

ii. $p(t) = 0$, $t_d \le t$

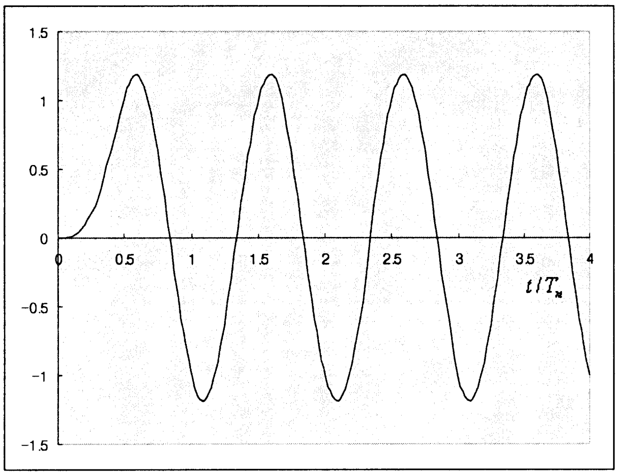
$$u(t) = u(t_d)\cos\omega_n(t-t_d) + \frac{\dot{u}(t_d)}{\omega_n}\sin\omega_n(t-t_d)$$

※ $u(t_d) = \frac{p_0}{k}\left[1 - \frac{1}{2\pi}\frac{T_n}{t_d}\sin\frac{2\pi t_d}{T_n}\right]$, $\dot{u}(t_d) = \frac{p_0}{k}\left[\frac{1}{t_d} - \frac{1}{t_d}\cos\frac{2\pi t_d}{T_n}\right]$ 대입하면,

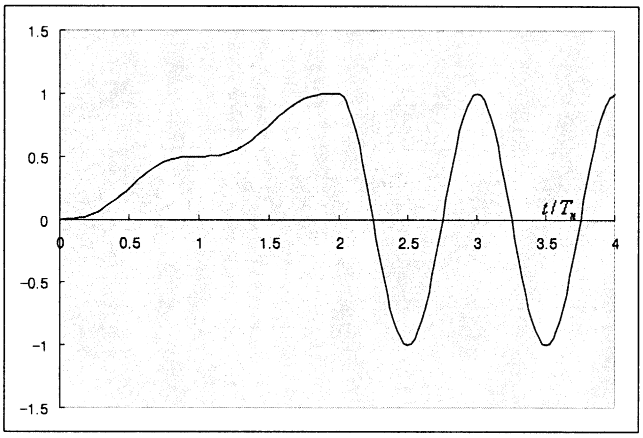
$$u(t) = \frac{p_0}{k}\left[\cos\frac{2\pi}{T_n}(t-t_d) + \frac{1}{2\pi}\frac{T_n}{t_d}\sin\frac{2\pi}{T_n}(t-t_d) - \frac{1}{2\pi}\frac{T_n}{t_d}\sin\frac{2\pi t}{T_n}\right]$$

plot response curve.

i. $t_d / T_n = \frac{1}{2}$



ii. $t_d / T_n = 2$

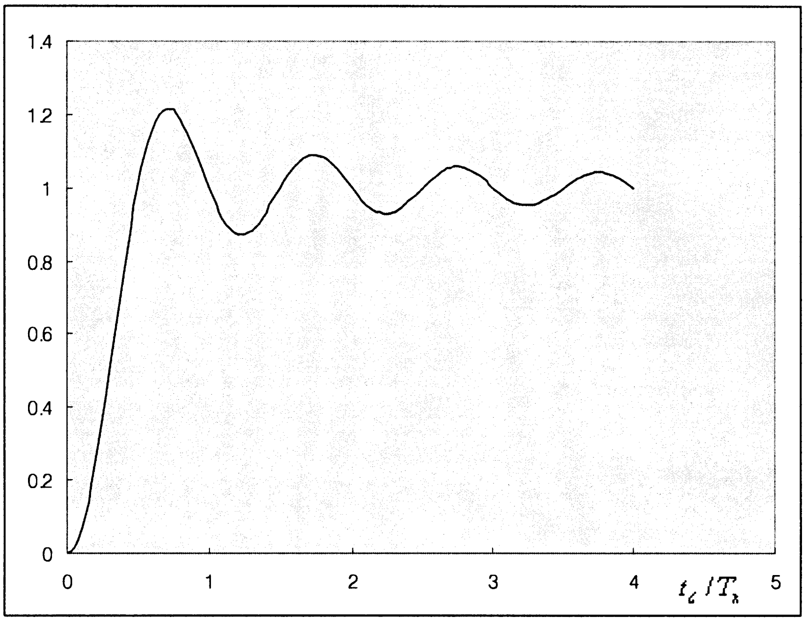



(b)&(c)

i. 강제진동단계

응답의 최대 변위 $u_0$는 $t = t_d$에서 발생.

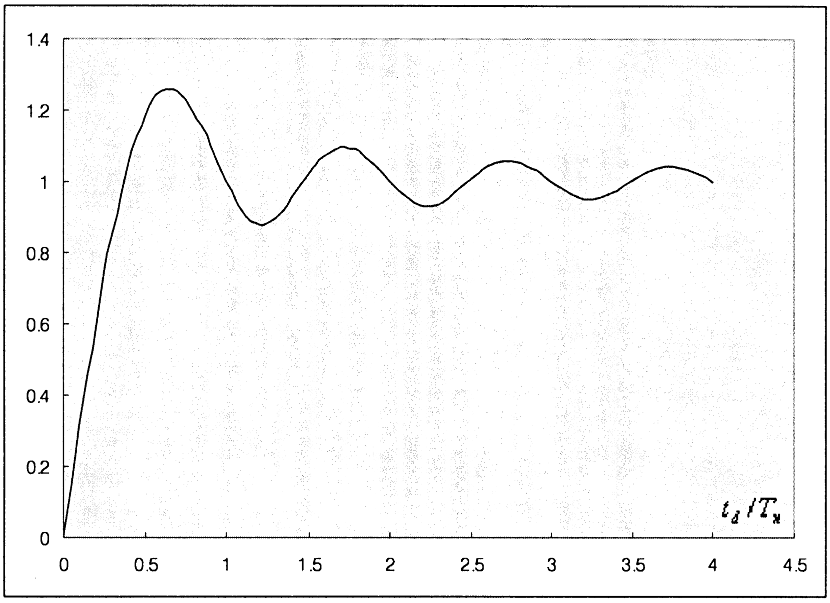
$$\therefore R_d = \frac{u(t_d)}{(u_{st})_0} = 1 - \frac{1}{2\pi}\frac{T_n}{t_d}\sin\frac{2\pi t_d}{T_n}$$



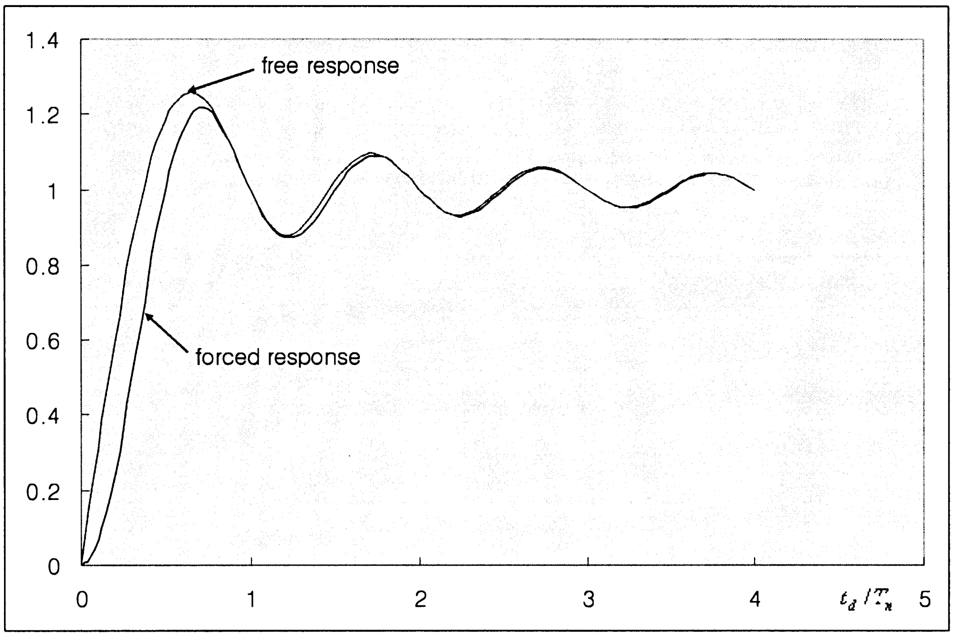
ii. 자유진동단계

$$u_0=\sqrt{u(t_d)^2+\left(\frac{\dot{u}(t_d)}{\omega_n}\right)^2}=\frac{p_0}{k}\sqrt{\left(1-\frac{1}{2\pi}\frac{T_n}{t_d}\sin\frac{2\pi t_d}{T_n}\right)^2+\frac{1}{t_d{}^2\omega_n{}^2}\left(1-\cos\frac{2\pi t_d}{T_n}\right)^2}$$

$$=\frac{p_0}{k}\sqrt{\left(1-\frac{1}{2\pi}\frac{T_n}{t_d}\sin\frac{2\pi t_d}{T_n}\right)^2+\frac{1}{4\pi^2}\left(\frac{T_n}{t_d}\right)^2\left(1-\cos\frac{2\pi t_d}{T_n}\right)^2}=(u_{st})_0R_d$$

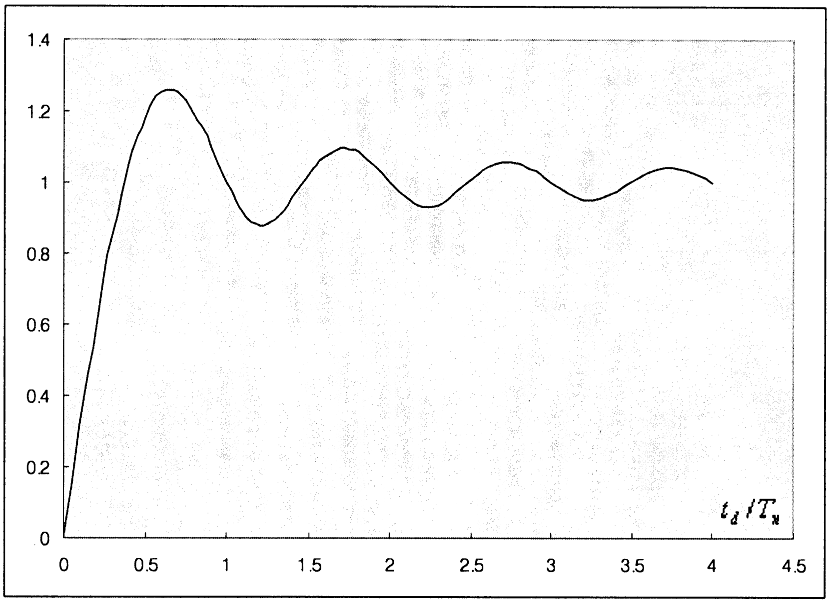



<종합>



<충격 스펙트럼>

free response일때가 항상 maximum값을 가진다.




# 4.25

(a)

ⅰ. $0 \le t \le \frac{t_d}{2}$, $p(t) = p_0$

$$\underline{u_1(t) = (u_{st})_0 (1 - \cos\omega_n t)}$$

ⅱ. $\frac{t_d}{2} \le t \le t_d$, $p(t) = -p_0$

$$u_2(t) = A\cos\omega_n\left(t - \frac{t_d}{2}\right) + B\sin\omega_n\left(t - \frac{t_d}{2}\right) - (u_{st})_0$$

$$u_2\left(\frac{t_d}{2}\right) = A - (u_{st})_0 = u_1\left(\frac{t_d}{2}\right) = (u_{st})_0\left(1 - \cos\frac{\omega_n t_d}{2}\right)$$

$$\therefore A = (u_{st})_0\left(2 - \cos\frac{\omega_n t_d}{2}\right)$$

$$\dot{u}_2\left(\frac{t_d}{2}\right) = B\omega_n = \dot{u}_1\left(\frac{t_d}{2}\right) = (u_{st})_0\,\omega_n \sin\left(\frac{\omega_n t_d}{2}\right)$$

$$\therefore B = (u_{st})_0 \sin\left(\frac{\omega_n t_d}{2}\right)$$

$$u_2(t) = (u_{st})_0 \left[ \left\{ 2 - \cos\frac{\omega_n t_d}{2} \right\} \cos\omega_n \left( t - \frac{t_d}{2} \right) + \sin\left( \frac{\omega_n t_d}{2} \right) \sin\omega_n \left( t - \frac{t_d}{2} \right) - 1 \right]$$

iii. $t_d \le t$, $p(t) = 0$

$$u_3(t) = u(t_d)\cos\omega_n(t - t_d) + \frac{\dot{u}(t_d)}{\omega_n}\sin\omega_n(t - t_d)$$

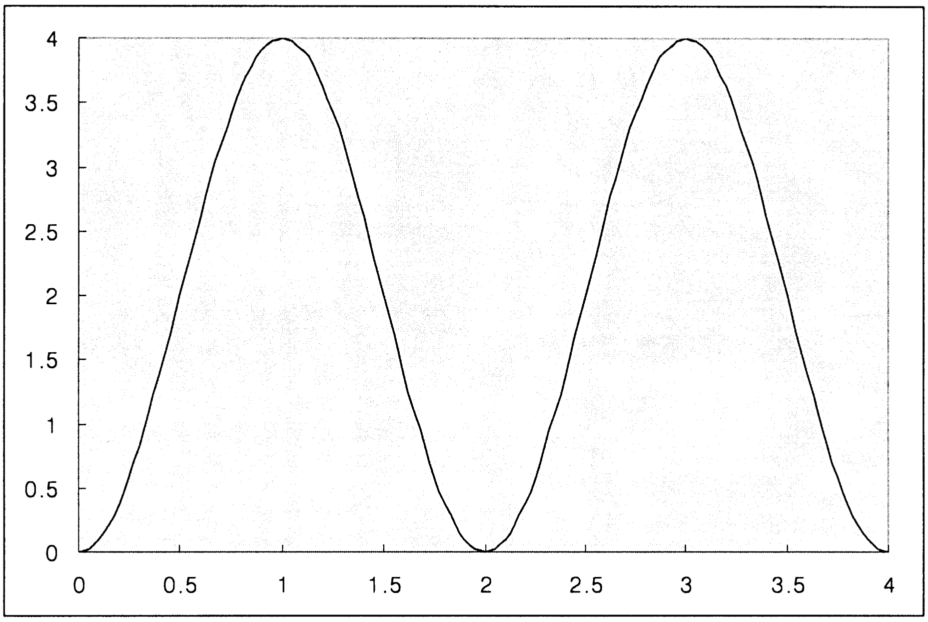
$$u(t_d) = u_2(t_d) = (u_{st})_0 \left[ 2\cos\frac{\omega_n t_d}{2} - \cos\omega_n t_d - 1 \right]$$

$$\dot{u}(t_d) = \dot{u}_2(t_d) = (u_{st})_0 \omega_n \left[ \sin\omega_n t_d - 2\sin\frac{\omega_n t_d}{2} \right],$$ 대입하면

$$u_3(t) = (u_{st})_0 \left[ \left\{ 2\cos\frac{\omega_n t_d}{2} - \cos\omega_n t_d - 1 \right\} \cos\omega_n(t - t_d) + \left\{ \sin\omega_n t_d - 2\sin\frac{\omega_n t_d}{2} \right\} \sin\omega_n(t - t_d) \right]$$

(b)

$$u_0 = \sqrt{u(t_d)^2 + \left( \frac{\dot{u}(t_s)}{\omega_n} \right)^2} = 4(u_{st})_0 \sin^2\left( \frac{\pi t_d}{2T_n} \right) = (u_{st})_0 R_d$$




(c) 주어진 하중을 pure impulse로 가정하고 풀면 impulse의 값이 0이 되어 무의미한 결과가 도출된다.