10.8

$$\mathbf{k} = \frac{24EI}{h^3}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{m} = m\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$\left[\mathbf{k} - \omega_n{}^2\mathbf{m}\right]\phi_n = \mathbf{0}$$

$$\det\left[\mathbf{k} - \omega_n{}^2\mathbf{m}\right] = 0, \quad \rightarrow \det\left[\frac{24EI}{h^3}\begin{bmatrix} 2-2\lambda & -1 \\ -1 & 1-\lambda \end{bmatrix}\right] = 0, \quad \lambda = \frac{h^3 m}{48EI}\omega_n{}^2$$

$$2\lambda^2 - 4\lambda - 1 = 0, \rightarrow \lambda_1 = 0.292895, \; \lambda_2 = 1.707105$$

$$\omega_1 = 9.05212\sqrt{\frac{EI}{mh^3}}, \quad \omega_2 = 3.74953\sqrt{\frac{EI}{mh^3}}$$

$$\phi_1 = \begin{Bmatrix} 0.7071 \\ 1 \end{Bmatrix}, \quad \phi_2 = \begin{Bmatrix} -0.7071 \\ 1 \end{Bmatrix}$$

(a)

$$\mathbf{u}(0) = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}, \quad \dot{\mathbf{u}}(0) = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$q_1(0) = \frac{\phi_1{}^{\mathrm{T}}\mathbf{m}\mathbf{u}(0)}{\phi_1{}^{\mathrm{T}}\mathbf{m}\phi_1} = 1.707, \quad q_2(0) = \frac{\phi_2{}^{\mathrm{T}}\mathbf{m}\mathbf{u}(0)}{\phi_2{}^{\mathrm{T}}\mathbf{m}\phi_2} = 0.293, \quad \dot{q}_1(0) = 0, \quad \dot{q}_2(0) = 0$$

$$\rightarrow q_1(t) = 1.707\cos\omega_n t, \quad q_2(t) = 0.293\cos\omega_n t$$

$$\mathbf{u}(t) = \phi_1 q_1(t) + \phi_2 q_2(t) = \begin{bmatrix} 1.207 \\ 1.707 \end{bmatrix}\cos\omega_1 t + \begin{bmatrix} -0.207 \\ 0.293 \end{bmatrix}\cos\omega_2 t$$

운동이 1차 모드를 나타내므로 1차 모드의 기여도가 더 크다.

(b)

$$\mathbf{u}(0) = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}, \quad \dot{\mathbf{u}}(0) = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$q_1(0) = \frac{\phi_1{}^{\mathrm{T}}\mathbf{m}\mathbf{u}(0)}{\phi_1{}^{\mathrm{T}}\mathbf{m}\phi_1} = -0.207, \quad q_2(0) = \frac{\phi_2{}^{\mathrm{T}}\mathbf{m}\mathbf{u}(0)}{\phi_2{}^{\mathrm{T}}\mathbf{m}\phi_2} = 1.207, \quad \dot{q}_1(0) = 0, \quad \dot{q}_2(0) = 0$$

$$\rightarrow q_1(t) = -0.207\cos\omega_1 t, \quad q_2(t) = 1.207\cos\omega_2 t$$

$$\mathbf{u}(t) = \phi_1 q_1(t) + \phi_2 q_2(t) = \begin{bmatrix} -0.146 \\ -0.207 \end{bmatrix}\cos\omega_1 t + \begin{bmatrix} -0.853 \\ 1.207 \end{bmatrix}\cos\omega_2 t$$

운동이 2차 모드를 나타내므로 2차 모드의 기여도가 더 크다.

10.16

$k_3 = 2\times\dfrac{12(EI/3)}{h^3} = \dfrac{8EI}{h^3} = k$ 라 하면, $k_1 = 3k$, $k_2 = 2k$

$$k_{i1} = \begin{Bmatrix} 5k \\ -2k \\ 0 \end{Bmatrix},\ \ k_{i2} = \begin{Bmatrix} -2k \\ 3k \\ -k \end{Bmatrix},\ \ k_{i3} = \begin{Bmatrix} 0 \\ -k \\ k \end{Bmatrix}\ \ \rightarrow \mathbf{k} = k\begin{Bmatrix} 5 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{Bmatrix}$$

$$\mathbf{m} = m\begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{Bmatrix}$$

$$\lfloor \mathbf{k} - \omega_n{}^2\mathbf{m} \rfloor \phi_n = 0,\quad \det\lfloor \mathbf{k} - \omega_n{}^2\mathbf{m} \rfloor = 0$$

$$\det\left[k\begin{Bmatrix} 5-\lambda & -2 & 0 \\ -2 & 3-\lambda & -1 \\ 0 & -1 & 1-0.5\lambda \end{Bmatrix} \right] = -\frac{\lambda^3}{2} + 5\lambda^2 - \frac{25\lambda}{2} + 6 = 0,\qquad \lambda = \frac{m\omega_n{}^2}{k}$$

$\lambda_1 = 0.6277$, $\lambda_2 = 3$, $\lambda_3 = 6.372$

$\omega_1 = 2.241\sqrt{\dfrac{EI}{mh^3}}$, $\omega_2 = 4.899\sqrt{\dfrac{EI}{mh^3}}$, $\omega_3 = 7.14\sqrt{\dfrac{EI}{mh^3}}$

$$\phi_1 = \begin{Bmatrix} 0.314 \\ 0.686 \\ 1 \end{Bmatrix},\ \ \phi_2 = \begin{Bmatrix} -0.5 \\ -0.5 \\ 1 \end{Bmatrix},\ \ \phi_3 = \begin{Bmatrix} 3.186 \\ -2.186 \\ 1 \end{Bmatrix}$$

$$q_1(0) = \frac{\phi_1{}^{\mathbf{T}}\mathbf{mu}(0)}{\phi_1{}^{\mathbf{T}}\mathbf{m}\phi_1},\quad q_2(0) = \frac{\phi_2{}^{\mathbf{T}}\mathbf{mu}(0)}{\phi_2{}^{\mathbf{T}}\mathbf{m}\phi_2},\quad q_3(0) = \frac{\phi_3{}^{\mathbf{T}}\mathbf{mu}(0)}{\phi_3{}^{\mathbf{T}}\mathbf{m}\phi_3}$$

$$q_n(t) = q_n(0)\cos\omega_n t + \frac{\dot{q}_n(0)}{\omega_n}\sin\omega_n t$$

$$\mathbf{u}(t) = \phi_1 q_1(t) + \phi_2 q_2(t) + \phi_3 q_3(t)$$

(a)

$$\mathbf{u}(0)=\begin{Bmatrix}1\\2\\3\end{Bmatrix},\quad \dot{\mathbf{u}}(0)=\begin{Bmatrix}0\\0\\0\end{Bmatrix}$$

$$q_1(0)=2.98,\quad q_2(0)=0,\quad q_3(0)=0.0205,\quad \dot{q}_1(0)=\dot{q}_2(0)=\dot{q}_3(0)=0$$
$$q_1(t)=2.98\cos\omega_1 t,\quad q_2(t)=0,\quad q_3(t)=0.0205\cos\omega_3 t$$

$$\mathbf{u}(t)=\phi_1 q_1(t)+\phi_2 q_2(t)+\phi_3 q_3(t)=\begin{bmatrix}0.935\\2.04\\2.98\end{bmatrix}\cos\omega_1 t+\begin{bmatrix}0.065\\-0.0668\\0.0205\end{bmatrix}\cos\omega_3 t$$

(b)

$$\mathbf{u}(0)=\begin{Bmatrix}-1\\0.25\\1\end{Bmatrix},\quad \dot{\mathbf{u}}(0)=\begin{Bmatrix}0\\0\\0\end{Bmatrix}$$

$$q_1(0)=0.334,\quad q_2(0)=0.875,\quad q_3(0)=-0.209,\quad \dot{q}_1(0)=\dot{q}_2(0)=\dot{q}_3(0)=0$$
$$q_1(t)=0.334\cos\omega_1 t,\quad q_2(t)=0.875\cos\omega_2 t,\quad q_3(t)=-0.209\cos\omega_3 t$$

$$\mathbf{u}(t)=\phi_1 q_1(t)+\phi_2 q_2(t)+\phi_3 q_3(t)=\begin{bmatrix}0.1048\\0.229\\0.334\end{bmatrix}\cos\omega_1 t+\begin{bmatrix}-0.4375\\-0.4375\\0.875\end{bmatrix}\cos\omega_2 t+\begin{bmatrix}-0.666\\0.456\\-0.2089\end{bmatrix}\cos\omega_3 t$$

(c)

$$\mathbf{u}(0)=\begin{Bmatrix}1\\-1\\1\end{Bmatrix},\quad \dot{\mathbf{u}}(0)=\begin{Bmatrix}0\\0\\0\end{Bmatrix}$$

$$q_1(0)=0.12,\quad q_2(0)=0.5,\quad q_3(0)=0.38,\quad \dot{q}_1(0)=\dot{q}_2(0)=\dot{q}_3(0)=0$$
$$q_1(t)=0.12\cos\omega_1 t,\quad q_2(t)=0.5\cos\omega_2 t,\quad q_3(t)=0.38\cos\omega_3 t$$

$$\mathbf{u}(t)=\phi_1 q_1(t)+\phi_2 q_2(t)+\phi_3 q_3(t)=\begin{bmatrix}0.038\\0.082\\0.12\end{bmatrix}\cos\omega_1 t+\begin{bmatrix}-0.25\\-0.25\\0.5\end{bmatrix}\cos\omega_2 t+\begin{bmatrix}1.21\\-0.83\\0.38\end{bmatrix}\cos\omega_3 t$$

8번과 마찬가지로 각각의 모드와 같은 형태의 진동모드가 상대적으로 많은 영향을 끼친다.

10.29

$m = 100/386 = 0.259 kip - s^2 / in$

$$\mathbf{m} = 0.259\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad \mathbf{k} = 326.32\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$\mathbf{k}\bar{\mathbf{x}}_{j+1} = \mathbf{m}\mathbf{x}_j$ 와 $\lambda^{j+1} = \dfrac{\bar{\mathbf{x}}^{\mathbf{T}}_{j+1}\mathbf{k}\bar{\mathbf{x}}_{j+1}}{\bar{\mathbf{x}}^{\mathbf{T}}_{j+1}\mathbf{m}\bar{\mathbf{x}}_{j+1}}$ 을 이용해 iteration해 나가면 된다.

$\mathbf{x}_1 = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$ 으로부터 시작하면

j	$\mathbf{x}_j$	$\bar{\mathbf{x}}_{j+1}$	λ^{j+1}	$\mathbf{x}_{j+1}$
1	$\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 0.0036 \\ 0.0063 \\ 0.0075 \end{Bmatrix}$	338.68	$\begin{Bmatrix} 0.7777 \\ 1.3826 \\ 1.6418 \end{Bmatrix}$
2	$\begin{Bmatrix} 0.7777 \\ 1.3826 \\ 1.6418 \end{Bmatrix}$	$\begin{Bmatrix} 0.0024 \\ 0.0041 \\ 0.0048 \end{Bmatrix}$	337.87	$\begin{Bmatrix} 0.7989 \\ 1.3894 \\ 1.6094 \end{Bmatrix}$
3	$\begin{Bmatrix} 0.7989 \\ 1.3894 \\ 1.6094 \end{Bmatrix}$	$\begin{Bmatrix} 0.0024 \\ 0.0041 \\ 0.0048 \end{Bmatrix}$	337.86	$\begin{Bmatrix} 0.8020 \\ 1.3899 \\ 1.6055 \end{Bmatrix}$

$\omega_1 = \sqrt{337.86} = 18.3809$ $\qquad \phi_1 = \begin{Bmatrix} 0.8020 \\ 1.3899 \\ 1.6055 \end{Bmatrix}$

10.31

$\breve{\mathbf{k}}\phi = \breve{\lambda}\mathbf{m}\phi$

when $\breve{\mathbf{k}} = \mathbf{k} - \mu\mathbf{m}$, $\breve{\lambda} = \lambda - \mu$

and same process by 역백터반복법

start with $\mathbf{x}_1 = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$

① first mode

$\mu = 300$ 으로 가정

j	$\mathbf{x}_j$	μ	$\overline{\mathbf{x}}_{j+1}$	$\lambda^{(j+1)}$	$\mathbf{x}_{j+1}$
1	$\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$	300	$\begin{Bmatrix} 0.0327 \\ 0.0569 \\ 0.0659 \end{Bmatrix}$	337.869	$\begin{Bmatrix} 0.7993 \\ 1.3891 \\ 1.6096 \end{Bmatrix}$
2	$\begin{Bmatrix} 0.7993 \\ 1.3891 \\ 1.6096 \end{Bmatrix}$	300	$\begin{Bmatrix} 0.0212 \\ 0.0367 \\ 0.0424 \end{Bmatrix}$	337.856	$\begin{Bmatrix} 0.8024 \\ 1.3900 \\ 1.6051 \end{Bmatrix}$

$\omega_1 = \sqrt{337.856} = 18.3809$ $\qquad \phi_1 = \begin{Bmatrix} 0.8024 \\ 1.3900 \\ 1.6051 \end{Bmatrix}$

② second mode

$\mu = 2000$ 으로 가정

j	$\mathbf{x}_j$	μ	$\bar{\mathbf{x}}_{j+1}$	$\lambda^{(j+1)}$	$\mathbf{x}_{j+1}$
1	$\begin{Bmatrix} 1 \\ 0 \\ -0.5 \end{Bmatrix}$	2000	$\begin{Bmatrix} 0.0016 \\ -0.0001 \\ -0.0016 \end{Bmatrix}$	2516.5	$\begin{Bmatrix} 1.5813 \\ -0.1405 \\ -1.6394 \end{Bmatrix}$
2	$\begin{Bmatrix} 1.5813 \\ -0.1405 \\ -1.6394 \end{Bmatrix}$	2000	$\begin{Bmatrix} 0.0031 \\ 0.0000 \\ -0.0030 \end{Bmatrix}$	2521.1	$\begin{Bmatrix} 1.6264 \\ 0.0171 \\ -1.5611 \end{Bmatrix}$
3	$\begin{Bmatrix} 1.6264 \\ 0.0171 \\ -1.5611 \end{Bmatrix}$	2000	$\begin{Bmatrix} 0.0031 \\ 0.0000 \\ -0.0031 \end{Bmatrix}$	2521.7	$\begin{Bmatrix} 1.6011 \\ -0.0106 \\ -1.6126 \end{Bmatrix}$

$\omega_2 = \sqrt{2521.7} = 50.2167$ $\qquad$ $\phi_2 = \begin{Bmatrix} 1.6011 \\ -0.0106 \\ -1.6126 \end{Bmatrix}$

③ third mode

$\mu = 4000$ 으로 가정

j	$\mathbf{x}_j$	μ	$\bar{\mathbf{x}}_{j+1}$	$\lambda^{(j+1)}$	$\mathbf{x}_{j+1}$
1	$\begin{Bmatrix} 1 \\ -1 \\ 1 \end{Bmatrix}$	4000	$\begin{Bmatrix} 0.0006 \\ -0.0015 \\ 0.0020 \end{Bmatrix}$	4669.7	$\begin{Bmatrix} 0.5813 \\ -1.3977 \\ 1.7735 \end{Bmatrix}$
2	$\begin{Bmatrix} 0.5813 \\ -1.3977 \\ 1.7735 \end{Bmatrix}$	4000	$\begin{Bmatrix} 0.0013 \\ -0.0019 \\ 0.0021 \end{Bmatrix}$	4697.6	$\begin{Bmatrix} 0.9010 \\ -1.3837 \\ 1.5084 \end{Bmatrix}$
3	$\begin{Bmatrix} 0.9010 \\ -1.3837 \\ 1.5084 \end{Bmatrix}$	4000	$\begin{Bmatrix} 0.0011 \\ -0.0020 \\ 0.0023 \end{Bmatrix}$	4703.9	$\begin{Bmatrix} 0.7550 \\ -1.3901 \\ 1.6503 \end{Bmatrix}$
4	$\begin{Bmatrix} 0.7550 \\ -1.3901 \\ 1.6503 \end{Bmatrix}$	4000	$\begin{Bmatrix} 0.0012 \\ -0.0020 \\ 0.0022 \end{Bmatrix}$	4705.3	$\begin{Bmatrix} 0.8248 \\ -1.3897 \\ 1.5827 \end{Bmatrix}$

$\omega_3 = \sqrt{4705.3} = 68.5952$ $\qquad \phi_3 = \begin{Bmatrix} 0.8248 \\ -1.3897 \\ 1.5827 \end{Bmatrix}$