

Homework2. Solution

1. (1),(2),(3) 각 문항 당 10점씩 실험의 신빙성, 실험목적과의 부합, 결과 도출을 판단하여 종합적으로 점수를 매김

2. (1) Kinematic F.S.B.C

$$F = z - \eta(\vec{x}, t) = 0 \quad (F(x, t)=0, \text{ geometric surface})$$

$$\begin{aligned} \frac{DF}{Dt} &= \frac{\partial}{\partial t}(z - \eta) + u \frac{\partial}{\partial x}(z - \eta) + w \frac{\partial}{\partial z}(z - \eta) = 0 \\ &= -\frac{\partial \eta}{\partial t} - u \frac{\partial \eta}{\partial x} + w = 0 \\ \therefore \frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x} - \frac{\partial \phi}{\partial z} &= 0 \end{aligned}$$

$\eta = \theta(\varepsilon)$, $\phi = \theta(\varepsilon)$ 이라 가정하고,

Taylor series를 통해 Small amplitude wave에 대한 linearization을 하면,

$$\left(\frac{\partial \eta}{\partial t} + \cancel{\frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x}} - \frac{\partial \phi}{\partial z} \right)_{z=0} + \eta \underbrace{\frac{\partial}{\partial z} \left(\frac{\partial \eta}{\partial t} + \cancel{\frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x}} - \cancel{\frac{\partial \phi}{\partial z}} \right)}_{z=0} + \dots = 0$$

$$\therefore \frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} \quad \text{on } z=0 \quad (\text{Linearized K.F.S.B.C})$$

(2) Dynamic F.S.B.C

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + \frac{P(\vec{x}, t)}{\rho} + g \eta = C(t)$$

Far field에서는 ($\phi \rightarrow 0$ and $\eta \rightarrow 0$), $C(t) = P_{atm}$ 이 된다.

$$\text{따라서, } \frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + \frac{P(\vec{x}, t) - P_{atm}}{\rho} + g \eta = 0$$

Taylor series를 통해 Small amplitude wave에 대한 linearization을 하면,

$\eta = \theta(\varepsilon)$, $\phi = \theta(\varepsilon)$

$$\left(\frac{\partial \phi}{\partial t} + \cancel{\frac{1}{2} \nabla \phi \cdot \nabla \phi} + \frac{P(\vec{x}, t) - P_{atm}}{\rho} + g \eta \right)_{z=0} + \eta \frac{\partial}{\partial z} \left(\cancel{\frac{\partial \phi}{\partial t}} \right)_{z=0} + \frac{\eta}{2} \frac{\partial}{\partial z^2} \left(\cancel{\frac{\partial \phi}{\partial t}} \right)_{z=0} + \dots = 0$$

$$\therefore \frac{\partial \phi}{\partial t} + \frac{\vec{P}(x,t) - P_{atm}}{\rho} + g\eta = 0 \quad \text{on } z=0 \quad (\text{Linearized K.F.S.B.C})$$

3. 1),2),3) A=1.0m

(1) Wave length 1m and filling depth 0.5m

x(z=0)	potential	u	w
0λ	-1.251859636	0	-7.836341802
0.25λ	-4.09101E-07	7.865664438	-2.56088E-06
0.5λ	1.251859636	5.14092E-06	7.836341802
0.75λ	1.2273E-06	-7.865664438	7.68263E-06

x(z=-h/4)	potential	u	w
0λ	-0.574824582	0	-3.547415878
0.25λ	-1.8785E-07	3.611728619	-1.15928E-06
0.5λ	0.574824582	2.36059E-06	3.547415878
0.75λ	5.63549E-07	-3.611728619	3.47783E-06

x(z=-h/2)	potential	u	w
0λ	-0.270975934	0	-1.561535821
0.25λ	-8.85536E-08	1.702591651	-5.10302E-07
0.5λ	0.270975934	1.1128E-06	1.561535821
0.75λ	2.65661E-07	-1.702591651	1.53091E-06

x(z=-h)	potential	u	w
0λ	-0.107993918	0	0
0.25λ	-3.52919E-08	0.678545658	0
0.5λ	0.107993918	4.43491E-07	0
0.75λ	1.05876E-07	-0.678545658	0

(2) Wave length 1m and filling depth 0.5m

x(z=0)	potential	u	w
0λ	-1.845180629	0	-5.316552671
0.25λ	-6.02996E-07	5.796804702	-1.73742E-06
0.5λ	1.845180629	3.78873E-06	5.316552671

0.75λ	1.80899E-06	-5.796804702	5.21227E-06
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x(z=-h/4)	potential	u	w
0λ	-1.307511315	0	-3.396427013
0.25λ	-4.27288E-07	4.107667088	-1.10994E-06
0.5λ	1.307511315	2.68473E-06	3.396427013
0.75λ	1.28186E-06	-4.107667088	3.32981E-06

x(z=-h/2)	potential	u	w
0λ	-0.974081168	0	-2.006838577
0.25λ	-3.18325E-07	3.060165604	-6.55825E-07
0.5λ	0.974081168	2.00009E-06	2.006838577
0.75λ	9.54974E-07	-3.060165604	1.96747E-06

x(z=-h)	potential	u	w
0λ	-0.735372632	0	0
0.25λ	-2.40316E-07	2.310240777	0
0.5λ	0.735372632	1.50995E-06	0
0.75λ	7.20948E-07	-2.310240777	0

(3) Wave length 10 m and filling depth 0.5m

x(z=0)	potential	u	w
0λ	-7.163965628	0	-1.369353304
0.25λ	-2.34115E-06	4.501251421	-4.47498E-07
0.5λ	7.163965628	2.94197E-06	1.369353304
0.75λ	7.02344E-06	-4.501251421	1.34249E-06

x(z=-h/4)	potential	u	w
0λ	-7.014727257	0	-1.019687949
0.25λ	-2.29238E-06	4.407482206	-3.33229E-07
0.5λ	7.014727257	2.88069E-06	1.019687949
0.75λ	6.87713E-06	-4.407482206	9.99686E-07

x(z=-h/2)	potential	u	w
0λ	-6.908781479	0	-0.676315774

0.25λ	-2.25775E-06	4.340914525	-2.21017E-07
0.5λ	6.908781479	2.83718E-06	0.676315774
0.75λ	6.77326E-06	-4.340914525	6.6305E-07

x(z=-h)	potential	u	w
0λ	-6.824415407	0	0
0.25λ	-2.23018E-06	4.287905769	0
0.5λ	6.824415407	2.80253E-06	0
0.75λ	6.69055E-06	-4.287905769	0

3) Linear dynamic pressure

$$P_d = -\rho \frac{\partial \phi}{\partial t} = -\rho g A \frac{\cosh k(z+h)}{\cosh kh} \cos k \cos wt,$$

t=T/4일 때 이므로 이 때의 linear dynamic pressure는 0이 된다.

4) Linear dynamic+ static pressure

$$P_d + P_s = 0 + \rho gz$$

$$Z=0 : 0 \text{N/m}^2$$

$$Z=-0.125 : 1226 \text{N/m}^2$$

$$Z=-0.25 : 2452 \text{N/m}^2$$

$$Z=-0.5 : 4905 \text{N/m}^2$$