Homework1. Solution

1. For potential flow

$$u=\frac{\partial\phi}{\partial x}=6ye^{3z}\cos x \quad \rightarrow \quad \phi=2ye^{3z}\sin 3x+f(y,z) \tag{1.1}$$

$$v=\frac{\partial\phi}{\partial y} \quad \rightarrow \quad \phi=2ye^{3z}\sin 3x+g(x,z) \tag{1.2}$$

(1.3)=(1.4)이므로

$$f(y,z)=g(x,z) \quad \rightarrow \quad f(z)=g(z)\equiv F(z)$$

$$\text{Continuity eq}: \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial w}{\partial z}=-\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}=-\left(-18ye^{3z}\sin 3x+\cancel{\frac{\partial^2 F}{\partial x^2}}\right)-\cancel{\frac{\partial^2\phi}{\partial y^2}}=0$$

$$\therefore w=6ye^{3z}\sin 3x+C$$

2.

(1)

$$Fr=\frac{V_{\text{ship}}}{\sqrt{gL_{\text{ship}}}}=\frac{V_{\text{model}}}{\sqrt{gL_{\text{model}}}}, \quad V_{\text{ship}}=Fr\times\sqrt{gL_{\text{ship}}}=10.8m/s$$

$$V_{\text{model}}=V_{\text{ship}}\times\sqrt{\frac{L_{\text{model}}}{L_{\text{ship}}}}=10.8\times\sqrt{\frac{1}{100}}=1.08m/s$$

(2)

$$Assume: \frac{F}{\rho L^2U^2}\approx f'(\frac{gL}{U^2})$$

$$\frac{F_{\text{ship}}}{\rho L_{\text{ship}}^2V_{\text{ship}}^2}=\frac{F_{\text{model}}}{\rho L_{\text{model}}^2V_{\text{model}}^2}$$

$$F_{\text{ship}}=F_{\text{model}}\frac{L_{\text{ship}}^2V_{\text{ship}}^2}{L_{\text{model}}^2V_{\text{model}}^2}=F_{\text{model}}\frac{L_{\text{ship}}^3}{L_{\text{model}}^3}=10kN\times 100^3=10^7\,kN$$

(3) $\text{Re}=\frac{VL}{\nu}=\frac{1.08\times 3}{1.13902\times 10^{-6}}=2.855\times 10^6 (15^{\circ}C,\ freshwater)$

(4) $Fr=\frac{V}{\sqrt{gL}}, \text{Re}=\frac{VL}{\nu}$

Fr수는 $\frac{V}{\sqrt{L}}$ 에 ,Re수는 VL 에 비례하므로 모형실험에서 실선과 모형선의 Fr를 맞혀주는 V는 적당하지만 Re를 맞혀주는 모형선의 V는 축적 비에 반비례하기 때문에 매우 빠르거나 유체를 물이 아닌 다른 것을 써서 kinematic viscosity를 변화시켜야 한다. 이는 현실적으로 거의 불가능하다.

3.

(1) Continuity eq : $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

(2) $\nabla\times\vec{V}=0$

(3) ideal fluid이고 irrotational flow이면 velocity potential, Φ을 정의할 수 있다.

$\nabla\cdot\vec{V}=0$, 이고 $\vec{V}=\nabla\cdot\Phi$ 이므로 $\nabla\cdot(\nabla\cdot\Phi)=\nabla^2\cdot\Phi=0$

4. (1) $\Phi=\Phi_1+\Phi_2=Ur\cos\theta+\frac{m}{2\pi}\ln r$

(2) $\Phi=Ux+\frac{m}{2\pi}\ln(x^2+y^2)^{1/2}$, (x,y)=(0,0) at source center

$$\nabla^2\Phi=\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0$$

$$\frac{\partial\Phi}{\partial x}=U+\frac{m}{2\pi}(x^2+y^2)^{-1}\cdot x=0$$

$$\frac{\partial^2\Phi}{\partial x^2}=\frac{m}{2\pi}(y^2-x^2)(x^2+y^2)^{-2}=0$$

$$\frac{\partial\Phi}{\partial y}=\frac{m}{2\pi}(x^2+y^2)^{-1}\cdot y=0$$

$$\frac{\partial^2\Phi}{\partial y^2}=\frac{m}{2\pi}(x^2-y^2)(x^2+y^2)^{-2}=0$$

$$\therefore\nabla^2\Phi=0$$

**다른 풀이

$$\nabla^2\Phi=\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial\Phi}{\partial r}+\frac{m}{2\pi r^2}$$

$$\frac{\partial^2\Phi}{\partial r^2}=-\frac{m}{2\pi r^2}$$

$$\frac{1}{r}\cdot\frac{\partial\Phi}{\partial r}=\frac{1}{r}\left(U\cos\theta+\frac{m}{2\pi r}\right)=\frac{U}{r}\cos\theta+\frac{m}{2\pi r^2}$$

$$\frac{m}{2\pi r^2}=\frac{1}{r^2}\left(-Ur\cos\theta\right)=-\frac{U}{r}\cos\theta$$

$$\nabla^2\Phi=-\frac{m}{2\pi r^2}+\frac{U}{r}\cos\theta+\frac{m}{2\pi r^2}-\frac{U}{r}\cos\theta=0$$

$$\therefore\nabla^2\Phi=0$$

(3)

$$\Phi=Ux+\frac{m}{2\pi}\ln(x^2+y^2)^{1/2}$$

$$\frac{\partial\Phi}{\partial y}=\frac{m}{4\pi}(x^2+y^2)^{-1}\cdot 2y=0 \qquad \Rightarrow\ y=0$$

$$\frac{\partial\Phi}{\partial x}=U+\frac{m}{4\pi}(x^2+y^2)^{-1}\cdot 2x=0$$

위 식에 y=0을 대입하면

$$\frac{\partial\Phi}{\partial x}=U+\frac{m}{2\pi}(x)^{-1}=0$$

$$x=-\frac{m}{2\pi U}$$

$$\therefore \text{stagnation point}=\left(-\frac{m}{2\pi U},0\right)$$

(4)Bernoulli's Eq을 사용하면,

$$p_s+\frac{1}{2}\rho\vec{V_s}^2+\rho gz=p_\circ+\frac{1}{2}\rho\vec{U}^2+\rho gz$$

Stagnation point에서의 속도, $\vec{V_s}$ =0이므로

$$p_s=p_\circ+\frac{1}{2}\rho\vec{U}^2$$ 이다.

5.

Euler equations

$$X-\frac{1}{\rho}\frac{\partial p}{\partial x}=u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\frac{\partial u}{\partial t} \qquad \cdots\cdots①$$

$$Y-\frac{1}{\rho}\frac{\partial p}{\partial y}=u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\frac{\partial v}{\partial t} \qquad \cdots\cdots②$$

$$Z-\frac{1}{\rho}\frac{\partial p}{\partial z}=u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}+\frac{\partial w}{\partial t} \qquad \cdots\cdots③$$

Assumptions

ii) Incompressible and irrotational flow

$$\nabla\phi=\left(\frac{\partial\phi}{\partial x},\ \frac{\partial\phi}{\partial y},\ \frac{\partial\phi}{\partial z}\right)=(u,\ v,\ w)$$

$$\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x},\quad \frac{\partial u}{\partial z}=\frac{\partial w}{\partial x},\quad \frac{\partial v}{\partial z}=\frac{\partial w}{\partial y}$$

i) The extraneous force components are given by

$$X=-\frac{\partial\Omega}{\partial x},\quad Y=-\frac{\partial\Omega}{\partial y},\quad Z=-\frac{\partial\Omega}{\partial z}.$$

① => $$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial x}+w\frac{\partial w}{\partial x}+\frac{\partial^2\phi}{\partial t\partial x}+\frac{\partial\Omega}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}u^2+\frac{1}{2}v^2+\frac{1}{2}w^2+\frac{\partial\phi}{\partial t}+\Omega+\frac{p}{\rho}\right)=0$$

Integrate with respect to x

$$\frac{\partial\phi}{\partial t}+\frac{1}{2}(u^2+v^2+w^2)+\Omega+\frac{p}{\rho}=F_1(y,z,t)$$

② => $$\frac{\partial\phi}{\partial t}+\frac{1}{2}(u^2+v^2+w^2)+\Omega+\frac{p}{\rho}=F_2(x,z,t)$$

③ => $$\frac{\partial\phi}{\partial t}+\frac{1}{2}(u^2+v^2+w^2)+\Omega+\frac{p}{\rho}=F_3(x,y,t)$$

$$\therefore \underline{\frac{\partial\phi}{\partial t}+\frac{1}{2}|\nabla\phi|^2+\Omega+\frac{p}{\rho}=F(t)}$$

● gravity is the only extraneous force acting,

$$\Omega=gz$$

$$\therefore \frac{1}{2}|\nabla\phi|^2+\frac{\partial\phi}{\partial t}+gz+\frac{p}{\rho}=F(t)$$

● steady flow인 경우

$$\therefore \frac{1}{2}|\nabla\phi|^2+gz+\frac{p}{\rho}=C \qquad (C:\text{constant})$$