

Homework 5

Course: 414.311A

Due Dec. 8

1. ISSC spectrum can be written as follows:

$$S_{\eta}(\omega) = \frac{A}{\omega^5} e^{-B/\omega^4}$$

where $A = \frac{173H_{1/3}^2}{T_1^4}$, $B = \frac{691}{T_1^4}$, $H_{1/3}$ is significant wave height, and T_1 is mean wave

period. Lets' consider ocean waves in which $H_{1/3}$ and T_1 are 6.0m and 8sec.

(1) Discretize this wave spectrum into 49 components in a frequency range of 0.1~5rad/sec.

Find the wave amplitudes of 49 components.

(2) Find the mean wave energy density of this ocean.

(3) Generate a time-history of wave elevation at a fixed point in a time range of 100sec.

(Remember that phase difference of each component is random and has a uniform probability.)

2. Consider a spar buoy as shown right. Your task is to decide the freeboard of this platform at Hurricane condition. Assume that ISSC spectrum is valid in this region.

(1) If the significant wave height is 9m and the modal period is 20sec (this condition corresponds sea state 9), find the wave height of probability of 1%. (note: The modal period and mean period are not the same.)

(2) If this ocean is represented by 4.0m significant wave

height and 10sec modal wave period, find the wave height of 100-year return period, which waves cannot reach to the deck. (This is an example of short-term prediction.)

(3) The following table summarizes the number of occurrence of significant wave height and modal (peak) period, which was observed in this ocean. The total number of measurement is 800, and the probability function of wave elevation showed the Gaussian (normal) distribution.



To design the height of freeboard, you need to know the wave height of return period 100 and 200 years. Obtain the design wave heights. You need to show your procedure of prediction. (This is an example of long-term prediction.)

| $H_{1/3}$ | $T_{\text{peak}} (=T_{\text{modal}})$ | | | | | | | | | | |
|-----------|---------------------------------------|----|-----|-----|-----|-----|----|----|----|----|-----|
| | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | sum |
| 2 | 8 | 22 | 30 | 31 | 24 | 22 | 17 | 11 | 5 | 2 | 172 |
| 4 | 4 | 35 | 42 | 45 | 43 | 38 | 28 | 17 | 11 | 4 | 267 |
| 6 | 0 | 15 | 28 | 24 | 31 | 28 | 21 | 15 | 10 | 3 | 175 |
| 8 | 0 | 1 | 19 | 17 | 22 | 21 | 17 | 11 | 6 | 2 | 116 |
| 10 | 0 | 0 | 1 | 8 | 12 | 13 | 13 | 6 | 2 | 1 | 56 |
| 12 | 0 | 0 | 0 | 0 | 2 | 4 | 2 | 1 | 1 | 0 | 10 |
| 14 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 4 |
| sum | 12 | 73 | 120 | 125 | 134 | 127 | 99 | 62 | 36 | 12 | 800 |

Table 1. Joint frequency of significant wave height and modal period