Aeroelasticity Assignment No. 2

Due Date: October 30 (Fri) 6:00 PM

1. Consider an airfoil subject to pitch (α) and plunge (h) motions as shown in Fig. 1.

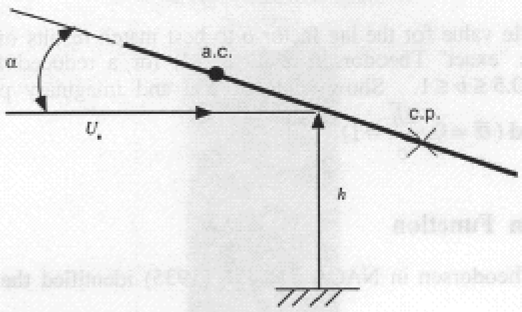

Figure 1. Airfoil in motion

Also, consider the lumped-vortex model of Fig. 2, where the normalwash at the control point is given by

$$w_{cp} = -\frac{\Gamma(t)}{\pi c} - \frac{b}{U_\infty}\frac{d\Gamma(t)}{dt}$$

where b is the *lag factor* (empirical).

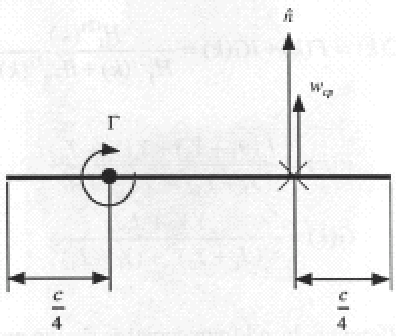

Figure 2. Lumped-Vortex Model

a) Apply flow tangency at control point, and establish the ordinary differential equation for $\Gamma(t)$ in terms of α, $\dot{\alpha}$, and $\dot{h}$.

b) Assume simple harmonic motion $\alpha = \bar{\alpha} e^{i\omega t}$, $h = \bar{h} e^{i\omega t}$, determine the "response" $\Gamma = \bar{\Gamma} e^{i\omega t}$.

c) Determine lift $L = \bar{L} e^{i\omega t}$ from the lumped-vortex result $L = \rho U_\infty \Gamma + \rho c \frac{d\Gamma}{dt}$ (from unsteady Bernoulli).

d) Determine $\bar{L}$ from Theodorsen's result.

Refer to Prof. Drela's note (posted at eTL), p.9, "Typical-Section Flutter Analysis."
Be careful about the definition of the reduced frequency (k) between in the reference and in the previous step.

e) What is a suitable value for the lag factor b to best match results of the lumped-vortex model (c) with the "exact" Theodorsen result of (d) for a reduced frequency range of $0 \le k \le 10$? Try $0.5 \le b \le 1$. Show plots of real and imaginary parts of the lift for ($\bar{\alpha} = 1, \frac{2\bar{h}}{c} = 0$) and ($\bar{\alpha} = 0, \frac{2\bar{h}}{c} = 1$).

Before plotting the lift component, compare the formulas obtained in (c) and (d), especially regarding the part for $C(k)$. (You may need to consider only the circulatory part in the result of (d).)
Then, predict an appropriate value for "b" by taking a limit of k to infinity.
With this prediction of b, you may now plot the lift component corresponding to (c) and (d), with and without the non-circulatory part (apparent mass term).

Note: **Theodorsen Function**

Theodore Theodorsen in NACA TR 496 (1935) identified the following ratio of integrals:

$$C(k)=\frac{\int_1^\infty \frac{x}{\sqrt{x^2-1}}e^{-ikx}dx}{\int_1^\infty \frac{x+1}{\sqrt{x^2-1}}e^{-ikx}dx}$$

as the fundamental function of reduced frequency (k), which describes the wake influence on the unsteady circulatory airloads. He also showed that the integral ratio could be written in terms of Hankel functions of the 2nd kind, or Bessel functions of the 1st and 2nd kinds as

$$C(k)=F(k)+iG(k)=\frac{H_1^{(2)}(k)}{H_1^{(2)}(k)+iH_0^{(2)}(k)}$$

and

$$F(k)=\frac{J_1(J_1+Y_0)+Y_1(Y_1+J_0)}{(J_1+Y_0)^2+(Y_1-J_0)^2}$$

$$G(k)=-\frac{Y_1Y_0+J_1J_0}{(J_1+Y_0)^2+(Y_1-J_0)^2}$$

(A table with values for different reduced frequencies is shown on the next page)

Theodorsen Function

$C(k) = F(k) + i\,G(k)$

Ref.: AGARD Manual on Aeroelasticity, Vol. VI

k	F(k)	-G(k)	k	F(k)	-G(k)
0	1.0000000	0.0000000	0.64	0.5726853	0.1330545
0.01	0.9824216	0.0456521	0.66	0.5698898	0.1307822
0.02	0.9637253	0.0752079	0.68	0.5672518	0.1285708
0.03	0.9450111	0.0979135	0.70	0.5647596	0.1264189
0.04	0.9267018	0.1160013	0.72	0.5624026	0.1243252
0.05	0.9090087	0.1306443	0.74	0.5601712	0.1222882
0.06	0.8920397	0.1425944	0.76	0.5580567	0.1203065
0.08	0.8604318	0.1604021	0.78	0.5560509	0.1183784
0.10	0.8319241	0.1723022	0.80	0.5541466	0.1165024
0.12	0.8063273	0.1800727	0.82	0.5523369	0.1146768
0.14	0.7833715	0.1848904	0.86	0.5489774	0.1111714
0.16	0.7627719	0.1875659	0.90	0.5459286	0.1078496
0.18	0.7442570	0.1886727	0.94	0.5431533	0.1046996
0.20	0.7275799	0.1886242	0.98	0.5406197	0.1017105
0.22	0.7125211	0.1877232	1.00	0.5394349	0.1002729
0.24	0.6988879	0.1861940	1.10	0.5342148	0.0936062
0.26	0.6865125	0.1842043	1.20	0.5299560	0.0877090
0.28	0.6752492	0.1818807	1.30	0.5264367	0.0824643
0.30	0.6649711	0.1793191	1.40	0.5234957	0.0777759
0.32	0.6555686	0.1765929	1.50	0.5210132	0.0735641
0.34	0.6469460	0.1737580	1.60	0.5188992	0.0697629
0.36	0.6390200	0.1708575	1.70	0.5170845	0.0663173
0.38	0.6317179	0.1679244	1.80	0.5155155	0.0631816
0.40	0.6249763	0.1649840	1.90	0.5141501	0.0603171
0.42	0.6187392	0.1620556	2.00	0.5129548	0.0576913
0.44	0.6129575	0.1591543	2.10	0.5119026	0.0552762
0.46	0.6075879	0.1562909	2.20	0.5109717	0.0530482
0.48	0.6025921	0.1534740	2.30	0.5101443	0.0509871
0.50	0.5979361	0.1507095	2.40	0.5094058	0.0490750
0.52	0.5935896	0.1480019	2.50	0.5087440	0.0472969
0.54	0.5895258	0.1453541	3.00	0.5062799	0.0400039
0.56	0.5857205	0.1427682	4.00	0.5036709	0.0304961
0.58	0.5821522	0.1402450	5.00	0.5023973	0.0245986
0.60	0.5788016	0.1377852	10.00	0.5006178	0.0124467
0.62	0.5756512	0.1353885	∞	0.5000000	0