Homework # 2

(Due March 20, 2008)

1. Determine the positive-definiteness of the following quadratic forms:

   (a) $f = 2x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1 x_2 - 4x_2 x_3 \quad \mathbf{x} \in \mathrm{R}^3$

   (b) $f = 2x_1^2 + 5x_2^2 + 3x_3^2 + 8x_1 x_2 - 11x_2 x_3 + 2x_1 x_3 \quad \mathbf{x} \in \mathrm{R}^3$

   * Check by eigenvalues and Sylvester's test. (If needed, you can use MATLAB.)

2. Check the linear independence of the following vectors.

   (a) $\begin{Bmatrix} 1 \\ -5 \\ 2 \end{Bmatrix}, \begin{Bmatrix} 0 \\ 6 \\ 1 \end{Bmatrix}, \begin{Bmatrix} 3 \\ 3 \\ 3 \end{Bmatrix}$ (b) $\begin{Bmatrix} 2 \\ 1 \\ 1 \end{Bmatrix}, \begin{Bmatrix} 0 \\ 1 \\ -3 \end{Bmatrix}, \begin{Bmatrix} 2 \\ 3 \\ -5 \end{Bmatrix}$

3. If $f = \mathbf{g}^T \mathbf{h}$ where $f : R^n \to R^1, \mathbf{g} : R^n \to R^m, \mathbf{h} : R^n \to R^m$, show that

$$\nabla f = [\nabla \mathbf{h}]\mathbf{g} + [\nabla \mathbf{g}]\mathbf{h}$$

$$\boldsymbol{g} = [g_1(x_1, x_2, \ldots, x_n), g_2(x_1, x_2, \ldots, x_n), \ \ldots \ , g_m(x_1, x_2, \ldots, x_n)]^T$$

$$\boldsymbol{h} = [h_1(x_1, x_2, \ldots, x_n), h_2(x_1, x_2, \ldots, x_n), \ \ldots \ , h_m(x_1, x_2, \ldots, x_n)]^T$$

$$f = [g_1, g_2, \ldots, g_m][h_1, h_2, \ldots, h_m]^T \quad (scalar)$$