Homework # 4 (Due April 3, 2008)

1. For the function given, determine (a) all stationary points and (b) check whether the stationary points that you have obtained are strict local minima, using the sufficiency conditions; (taken from Problem 3.2 of Belegundu's book)

   (i) $f = 3x_1 + \dfrac{100}{x_1 x_2} + 5x_2$

   (ii) $f = (x_1 - 1)^2 + x_1 x_2 + (x_2 - 1)^2$

   (iii) $f = \dfrac{x_1 + x_2}{3 + x_1{}^2 + x_2{}^2 + x_1 x_2}$

2. Consider $f = 4x_1{}^2 + 3x_2{}^2 - 4x_1 x_2 + x_1$ with an initial search point $\mathbf{x}_0 = (-1/8, 0)^T$ and the direction vector $\mathbf{d}_0 = -(1/5, 2/5)^T$. [Same as Problem 3.3 of Belegundu's book]

   (i) is $\mathbf{d}_0$ a descent direction?

   (ii) Denoting $\hat{f}(\alpha) = f(\mathbf{x}_0 + \alpha \mathbf{d}_0)$, find $d\hat{f}(1)/d\alpha$.

3. For the function f in Problem 2, determine $\mathbf{x}_1$ by using the steepest descent method. Use $\mathbf{x}_0 = (-1, 0)^T$ as an initial guess. (Do not write an Matlab code for this problem. Solve it by hand calculation or by a calculator if necessary.)

4. Let ***A*** be a positive definite symmetric matrix. Show that any two eigenvectors of ***A***, corresponding to distinct eigenvalues, are conjugate with respect to ***A***.