Homework # 6 (Due May 8-Thursday, 2008)

1. Solve the following problem.

$$\text{minimize} \quad f(x_1, x_2) = 2x_1 + x_2 + 10$$
$$\text{subject to} \quad h(x_1, x_2) = x_1 + 2x_2^2 - 3 = 0$$

a) Find Lagrange points

b) Check if Lagrange point(s) is(are) minimum point(s). Calculate the objective function value f^* at the point(s).

c) If $h(x)$ is changed to $h_1(x) = x_1 + 2x_2^2 - 2.5 = 0$, the f^* in (b) will be also changed. To find f^* corresponding to $h_1(x)$, i) resolve the problem using the Lagrange method and ii) use the sensitivity analysis for approximate evaluation. Compare the two results.

2. We wish to solve the minimization problem by using the KKT condition.

$$\min f(x_1, x_2) = -x_1^3 - 2x_2^2 + 10x_1 - 6 + 2x_2^3$$
$$g_1 = x_1 - x_2 \le 0$$
$$g_2 = -x_1 \le 0$$
$$g_3 = x_2 - 10 \le 0$$

a) Write down the KKT condition

b) Using the KKT conditions for (a), find the minimum point(s) and evaluate the function value f^* at the points.

c) If the objective function value calculated in point(s) should be further reduced, which one of the constraints g_1, g_2 and g_3 must be relaxed? Why?

3. Solve the following problem by using the KKT condition.

$$\text{minimize} \quad f(\mathbf{x}) = x_1^2 + x_2^2 - 4x_1 - 6x_2$$
$$\text{subject to} \quad g_1(\mathbf{x}) = x_1 + x_2 - 2 \le 0$$
$$g_2(\mathbf{x}) = 2x_1 + 3x_2 - 12 \le 0$$