Homework 08_4 (Due: 3/31)

1. The binomial random variable X has PMF

    $$P_X(x) = \binom{5}{x}\left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}.$$

    (1) Find the standard deviation of X.

    (2) Find $P\left[\mu_X - \sigma_X \leq X \leq \mu_X + \sigma_X\right]$ which is the probability that X is within

    one standard deviation of the expected value.

    (3) Find $P_{X|B}(x)$, where the condition $B = \{X \geq \mu_X\}$.

    (4) Find $E[X \mid B]$ and $Var[X \mid B]$.

2. Prove Theorem 2.15 in the text.

3. A random variable X has PDF

    $$f_X(x) = \begin{cases} c(1-x^4) & -1 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

    (1) Find c.
    (2) Find the CDF of X.
    (3) Find $P\left[|X| < \frac{1}{2}\right]$.

4. Text Problem 2.10.5