Homework 08_6 (Due: 4/21)

1. Discrete random variables *X* and *Y* have the following joint PMF

$$P_{X,Y}(x,y)=\begin{cases}k(x^2+y^2) & x=-1,0,1;\ y=-2,0,2,\\ 0 & \text{otherwise.}\end{cases}$$

(1) What is the value of the constant *k*?

(2) What is $P[Y<X]$?

(3) Find the marginal PMF $P_X(x)$ and the expected value $E[X]$.

(4) Find the correlation coefficient, $\rho_{X,Y}$.

(5) Find the PMF $P_Z(z)$ and the expected value $E[Z]$ when *Z* is given by $Z=2^{XY}$.

2. Random variables *X* and *Y* have the following joint PMF

$$f_{X,Y}(x,y)=\begin{cases}4xy & 0\le y\le x\le 1,\\ 0 & \text{otherwise.}\end{cases}$$

(1) Find the joint CDF $F_{X,Y}(x,y)$.

(2) Find the marginal PDF $f_X(x)$ and the expected value $E[X]$.

(3) Find $Var[X+Y]$.

(4) Find $f_Z(z)$ when $Z=\max(X,Y)$.

3. Random variables *X* and *Y* have the following joint PMF

$$f_{X,Y}(x,y)=\begin{cases}\dfrac{x^2}{2} & -1\le x\le 1;\ 0\le y\le x^2,\\ 0 & \text{otherwise.}\end{cases}$$

Let $A=\left\{Y\le\dfrac{1}{4}\right\}$.

(1) Find the conditional PDF $f_{X,Y|A}(x,y)$.

(2) Find $f_{Y|A}(y)$ and $E[Y\mid A]$.

(3) Find $f_{X|A}(y)$ and $E[X \mid A]$.

4. Let X and Y be independent exponential random variables with the parameters λ_x and λ_y, respectively. Find the PDF of $Z = |X - Y|$.

5. Text Problem 4.12.2.