Homework 08_10 (Due: 6/2)

1. Consider the following moving average processes:

$$Y_n = \frac{1}{2}\left(X_n + X_{n-1}\right), \quad X_0 = 0,$$

$$Z_n = \frac{2}{3}X_n + \frac{1}{3}X_{n-1}, \quad X_0 = 0.$$

   (1) Flip a coin 10 times to obtain a realization of a Bernoulli random process X_n. Find the resulting realizations of Y_n and Z_n.

   (2) Find the mean, variance, and covariance of Y_n and Z_n if X_n is a Bernoulli random process. Are the sample means of Y_n and Z_n in part (1) close to their respective means?

   (3) Find the PDF of Y_n and Z_n if the X_n are an iid sequence of zero-mean, unit-variance Gaussian random variables.

2. Messages arrive at a computer from two telephone lines according to independent Poisson processes of rates λ_1 and λ_2, respectively.

   (1) Find the probability that a message arrives first on line 1.

   (2) Find the PDF for the time until a message arrives on either line.

   (3) Find the PMF for $N(t)$, the total number of messages that arrive in an interval of length *t*.

   (4) Generalize the result of part (3) for the "merging" of *k* independent Poisson processes of rates $\lambda_1, \cdots, \lambda_k$, respectively:

$$N(t) = N_1(t) + \cdots + N_k(t).$$

3. Text Problem 10.6.3.

4. Let $Z(t) = X(t) - aX(t-s)$, where $X(t)$ is the Brownian motion process.

   (1) Find the PDF of $Z(t)$.

   (2) Find the expected value and autocovariance of $Z(t)$.

5. Text Problem 10.13.3.