

due 11월 13일

**Exercise 1.3.** Readers who wish to test their understanding of this chapter by a simple graphical exercise may try the following.

[1] The electrical conductivity tensor of a certain crystal has the following components referred to axes  $x_1, x_2, x_3$ :

$$[\sigma_{ij}] = \begin{bmatrix} 25 \times 10^7 & 0 & 0 \\ 0 & 7 \times 10^7 & -(3\sqrt{3}) \times 10^7 \\ 0 & -(3\sqrt{3}) \times 10^7 & 13 \times 10^7 \end{bmatrix}$$

in m.k.s. units ( $\text{ohm}^{-1}\text{m}^{-1}$ ). The axes are now transformed to a new set  $x'_1, x'_2, x'_3$  given by the following angles:

$$x'_1 O x_1 = 0^\circ, x'_2 O x_2 = 30^\circ, x'_2 O x_3 = 60^\circ, x'_3 O x_3 = 30^\circ$$

Draw up a table of the form (11) (p.9) for this transformation, and check that the sum of the squares of the  $a_{ij}$  in each row and column is 1.

[2] Determine the values of the components  $\sigma'_{ij}$ , and comment on the result obtained.

[3] Draw on the new axes  $x'_2, x'_3$  a section of the conductivity ellipsoid (representation quadric) in the plane  $x'_1 = 0$ , and notice that this is a principal section.

Insert the old axes  $x_2, x_3$  on the drawing.

[4] Draw a radius vector  $OP$  in the direction whose cosines referred to the old axes are  $(0, \frac{1}{2}, \sqrt{3}/2)$ . Measure the length of this radius vector and so find the electrical conductivity in this direction.

[5] Check the last result by using an analytical expression.

[6] Assume an electric field of 1 volt/m to be established in the direction  $OP$ . Calculate the components  $E_i$  along the  $x_i$  axes, and hence calculate the components of current density  $j_i$ .