

# Homework #6

**7 - 5 A conducting circular loop of a radius 0.1(m) is situated in the neighborhood of a very long power line carrying a 60(Hz) current, as shown in Fig. 6 - 49, with  $d = 0.15(m)$ . A milliammeter inserted in the loop reads 0.3(mA). Assume the total impedance of the loop including the milliammeter to be  $0.01(\Omega)$ .**

- a) Find the magnitude of the current in the power line.**
- b) To what angle about the horizontal axis should the circular loop be rotated in order to reduce the milliammeter reading to 0.2(mA)?**

**7 - 6 A suggested scheme for reducing eddy - current power loss in transformer cores with a circular cross section is to divide the cores into a large number of small insulated filamentary parts. As illustrated in Fig. 7 - 12, the section shown in part (a) is replaced by that in part (b) Assuming that  $B(t) = B_0 \sin \omega t$  and that  $N$  filamentary areas fill 95% of the original cross - sectional area, find**

- a) the average eddy - current power loss in the section of core of height  $h$  in Fig.7 - 12(a)**
- b) the total average eddy - current power loss in the  $N$  filamentary sections in Fig. 7 - 12(b)**

**The magnetic field due to eddy currents is assumed to be negligible. (Hint : First find the current and power dissipated in the differential circular ring section of height  $h$  and width  $dr$  at radius  $r$ .)**

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**7-8 In the d - c motor illustrated in Fig. 6 - 32 we noted that a current  $I$  sent through the loop in a magnetic field  $B$  produces a torque that makes the loop rotate. As the loop rotates, the amount of the magnetic flux linking with the loop changes, giving rise to an induced emf. Energy must be expended by an external electric source to counter this emf and establish the current in the loop. Prove that this electric energy is equal to the mechanical work done by the rotating loop.**

**7-18 In Eqs.(3 - 88) and (3 - 89) it was shown that for field calculations a polarized dielectric may be replaced by an equivalent polarization surface charge density  $\rho_{ps}$  and an equivalent polarization volume charge density  $\rho_p$ . Find the boundary conditions at the interface of two different media for**

- a) the normal component of  $P$ ,**
- b) the normal components of  $E$**

**in terms of free and equivalent polarization surface charge densities  $\rho_s$  and  $\rho_{ps}$ .**

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**7 - 22** For the assumed  $f(t)$  at  $R = 0$  in Fig. 7 - 15, sketch

- a)  $f(t - R / u)$  versus  $t$ ,
- b)  $f(t - R / u)$  versus  $R$  for  $t > T$ .

**7 - 27** It is known that the electric field intensity of a spherical wave in free space is

$$\mathbf{E} = \mathbf{a}_\theta \frac{E_0}{R} \sin\theta \cos(\omega t - kR).$$

Determine the magnetic field intensity  $\mathbf{H}$  and the value of  $k$ .

**7 - 29** For a source - free polarized medium where  $\rho = 0$ ,  $\mathbf{J} = 0$ ,  $\mu = \mu_0$ , but where is a volume density of polarization  $\mathbf{P}$ , a single vector potential  $\pi_e$  may be defined such that

$$\mathbf{H} = \mathbf{j}\omega\epsilon_0 \nabla^2 \times \pi_e.$$

- a) Express electric field intensity  $\mathbf{E}$  in terms of  $\pi_e$  and  $\mathbf{P}$ .
- b) show that  $\pi_e$  satisfies the nonhomogeneous Helmholtz's equation

$$\nabla^2 \pi_e + k_0^2 \pi_e = -\frac{\mathbf{P}}{\epsilon_0}$$

The quantity  $\pi_e$  is known as the electric Hertz potential.

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**7-30 Calculations concerning the electromagnetic effect of currents in a good conductor usually neglect the displacement current even at microwave frequencies.**

- a) Assuming  $\epsilon_r = 1$  and  $\sigma = 5.70 \times 10^7$  (S / m) for copper, compare the magnitude of the displacement current density with that of the conduction current density at 100 (GHz).**
- b) Write the governing differential equation for magnetic field intensity  $H$  in a source - free good conductor.**