[Exercises 1] Samples

1.2 Consider a single-input-single-output system described by the nth-order differential equation

$$y^{(n)} = g_1(t, y, \dot{y}, \cdots, y^{(n-1)}, u) + g_2(t, y, \dot{y}, \cdots, y^{(n-2)})\dot{u}$$

Where g_2 is a differentiable function of its arguments. With u as input and y as output, find a state model.

Hint: Take $x_n = y^{(n-1)} - g_2(t, y, \dot{y}, \cdots, y^{(n-2)})u$.

- Hint를 참고하여, state variable x를 정리하면,

$$\begin{aligned}
x_1 &= y - g_2(t, y^{(-1)})u \\
x_2 &= y^{(1)} - g_2(t, y)u \\
x_3 &= y^{(2)} - g_2(t, y, y^{(1)})u \\
&\vdots \\
x_n &= y^{(n-1)} - g_2(t, y, y^{(1)}, \cdots, y^{(n-2)})u
\end{aligned} \tag{1}$$

식 (1)에 주어진 $y^{(n)}$을 대입하면,

$$\begin{aligned}
x_1 &= g_1(t, y^{(-1)}, u) + g_2(t, y^{(-2)}) - g_2(t, y^{(-1)})u \\
x_2 &= g_1(t, y, u) + g_2(t, y^{(-1)}) - g_2(t, y)u \\
x_3 &= g_1(t, y, y^{(1)}, u) + g_2(t, y) - g_2(t, y, y^{(1)})u \\
&\vdots \\
x_n &= g_1(t, y, y^{(1)}, \cdots, y^{(n-2)}, u) + g_2(t, y, y^{(1)}, y^{(n-3)}) - g_2(t, y, y^{(1)}, \cdots, y^{(n-2)})u
\end{aligned} \tag{2}$$

식 (2)를 y에 대하여 편미분하면,

$$
\begin{aligned}
\dot{x}_1 &= g_1(t, y, u) + g_2(t, y^{(-1)}) - g_2(t, y)u - g_2(t, y)\dot{u} \\
&= g_1(t, y, u) + g_2(t, y^{(-1)}) - g_2(t, y)u \quad (\because \dot{u} = 0) \\
&= x_2 \\
\dot{x}_2 &= g_1(t, y, u) + g_2(t, y^{(-1)}) - g_2(t, y)u \\
&= g_1(t, y, y^{(1)}, u) + g_2(t, y) - g_2(t, y, y^{(1)})u - g_2(t, y)\dot{u} \\
&= g_1(t, y, y^{(1)}, u) + g_2(t, y) - g_2(t, y, y^{(1)})u \\
&= x_3 \\
&\qquad \vdots \\
\dot{x}_n &= g_1(t, y, y^{(1)}, \cdots, y^{(n-1)}, u) + g_2(t, y, y^{(1)}, y^{(n-2)}) - g_2(t, y, y^{(1)}, \cdots, y^{(n-1)})u
\end{aligned}
\tag{3}
$$

$\dot{x}_n = h(t, x_1, x_2, \cdots, x_n, u)$ 라고 하면,

x_n에 대한 식 수정.

state-model 은 식 (3)으로부터,

$$
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}, \quad \dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ \vdots \\ h(t, x_1, x_2, \cdots, x_n, u) \end{bmatrix}
$$
이고,

$y = x_1 + g_2(t, y^{(-1)})u$,

1.7 Figure 1.18 shows a feedback connection of a linear time-invariant system represented by the transfer function $G(s)$ and a nonlinear time-varying element defined by $z=\psi(t,y)$. The variables r, u, y, and z are vectors of the same dimension, and $\psi(t,y)$ is a vector-valued function. With r as input and y as output, find a state model.

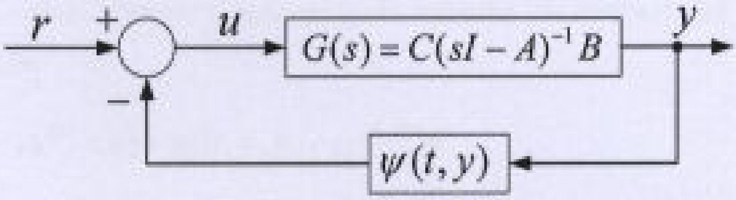
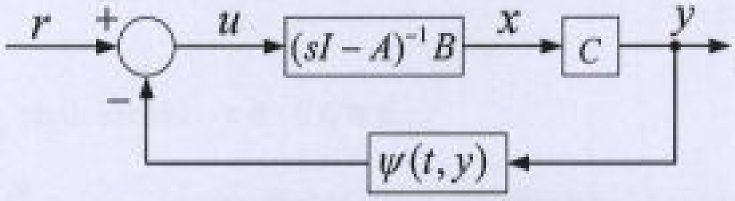


Figure 1.18

전달함수를 다음과 같이 분리하여, 블록다이어그램을 다시 그리면,



$u = r - \psi(t,y)y$

$\dot{x} = Ax + Bu$ 이므로,

$y = Cx$

* state model

$$\begin{cases} \dot{x} = f(t,x,u) \\ y = h(t,x,u) \end{cases}$$

$$\begin{aligned} \dot{x} &= Ax + B(r - \psi(t,y)y) \\ &= Ax - B\psi(t,y)y + Br \\ &= Ax - B\psi(t,Cx)Cx + Br \\ &= h(t,x,r) \end{aligned}$$

$y = Cx$

1.2) $y^{(n)} = g_1(t, y, \dot{y}, \cdots, y^{(n-1)}, u) + g_2(t, y, \dot{y}, \cdots, y^{(n-2)})\dot{u}$ — ⓪

u : input , y : output

Sol) $x_1 = y$ 두면

$$\begin{cases} x_2 = \dot{y} \\ x_3 = \ddot{y} \\ \vdots \\ x_{n-1} = y^{(n-2)} \\ x_n = y^{(n-1)} - g_2(t, y, \dot{y}, \cdots, y^{(n-2)})u \quad (\because \text{hint}) \quad — ① \end{cases}$$

↳ $y^{(n-1)} - g_2(t, x_1, x_2, \cdots, x_{n-1})u$

∴ state model 은

$\dot{x}_1 = \dot{y} = x_2$

$\dot{x}_2 = y^{(2)} = x_3$

$\vdots$

$\dot{x}_{n-1} = y^{(n-1)} = x_n + g_2(t, \underbrace{y, \dot{y}, \cdots, y^{(n-2)}}_{x_1, x_2, \cdots, x_{n-1}})u$ (∵ ① 식을 $y^{(n-1)}$에 대입)

$$\dot{x}_n = y^{(n)} - \frac{d}{dt}[g_2(t, x_1, x_2, \cdots, x_{n-1})]u - g_2(t, x_1, x_2, \cdots, x_{n-1})\dot{u}$$

$$= y^{(n)} - \left(\frac{\partial g_2}{\partial t}\underbrace{\frac{dt}{dt}}_{1} + \frac{\partial g_2}{\partial x_1}\underbrace{\frac{dx_1}{dt}}_{x_2} + \cdots + \frac{\partial g_2}{\partial x_{n-1}}\underbrace{\frac{dx_{n-1}}{dt}}_{\dot{x}_{n-1}}\right)u - g_2(t, x_1, \cdots, x_{n-1})\dot{u}$$

$$= y^{(n)} - \left(\frac{\partial g_2}{\partial t} + \frac{\partial g_2}{\partial x_1}\underbrace{\dot{x}_1}_{x_2} + \cdots + \frac{\partial g_2}{\partial x_{n-1}}\underbrace{\dot{x}_{n-1}}_{x_n + g_2(t, x_1, \cdots)u}\right)u - g_2(t, x_1, \cdots, x_{n-1}) \times \dot{u}$$

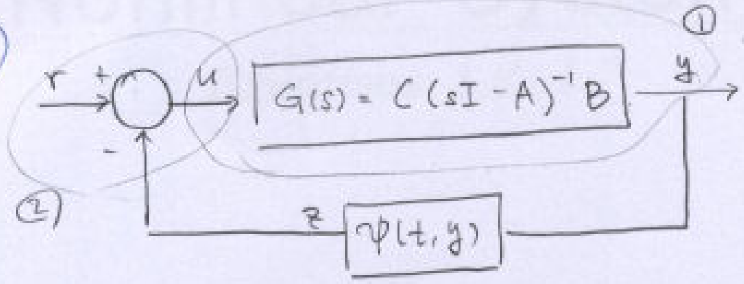
$y^{(n)}$에 ⓪식 대입

$$\therefore \dot{x}_n = g_1(t, x_1, x_2, \cdots, x_n + g_2 u) + g_2(t, x_1, \cancel{x_2}, \cdots, x_{n-1})\dot{u} - g_2(t, x_1, \cdots, x_{n-1})\dot{u} - \left(\frac{\partial g_2}{\partial t} + \frac{\partial g_2}{\partial x_1}x_2 + \cdots + \frac{\partial g_2}{\partial x_{n-1}}(x_n + g_2 u)\right)u$$

$\dot{x}_n = g_1(t, x_1, \cdots, x_n + g_2(t, x_1, x_2, x_{n-1})u, u)$ $y = x_1$

$$-\left[\frac{\partial g_2}{\partial t} + \frac{\partial g_2}{\partial x_1}x_2 + \cdots + \frac{\partial g_2}{\partial x_{n-1}}(x_n + g_2(t, x_1, x_2, \cdots, x_{n-1})u)\right]u$$

1.7) 5



G(s) : linear time-invariant

∴ ①의 부분은 다음과 같이 쓴다

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$ 위 sys. 에서는 $D = 0$.

∴ $\dot{x} = Ax + Bu$ — ㉠

$y = Cx$

②의 부분을 나타내면

$u = r - z = r - \psi(t, y) = r - \psi(t, Cx)$ — ⓐ

㉠에 ⓐ대입.

$\dot{x} = Ax + Br - B\psi(t, Cx)$

$$\therefore \begin{cases} \dot{x} = Ax - B\psi(t, Cx) + Br \\ y = Cx \end{cases}$$