[Exercises 5] Samples

\# 5.3. $y = u^{1/3} \Rightarrow h(u) = u^{1/3} \Rightarrow y = h(u) = Hu.$

(a) $\sup_{t\geq 0} |h(u(t))| \leq \left(\sup_{t\geq 0} |u(t)|\right)^{1/3}$

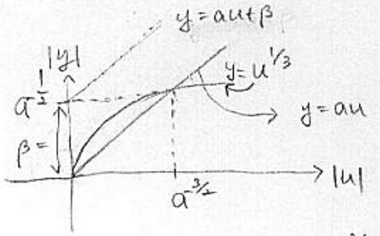
$\Rightarrow \|(Hu)_\tau\|_{\mathcal{L}_\infty} = \|y_\tau\|_{\mathcal{L}_\infty} \leq \left(\|u_\tau\|_{\mathcal{L}_\infty}\right)^{1/3}$

Let $\alpha(r) = r^{1/3}$, class K_∞ function defined on $[0, \infty)$

$\therefore \|(Hu)_\tau\|_{\mathcal{L}_\infty} \leq \alpha(\|u_\tau\|_{\mathcal{L}_\infty}) + \underset{\text{bias}}{0}$

$\therefore$ Def. 5.1 에 의해 system은 $\mathcal{L}_\infty$ stable with zero bias.

(b) We want $\|(Hu)_\tau\|_{\mathcal{L}_\infty} \leq \underset{\underset{a}{\parallel}}{\gamma} \|u_\tau\|_{\mathcal{L}_\infty} + \underset{\underset{(1/a)^{1/2}}{\parallel}}{\beta}$.



$u^{1/3} = au \Rightarrow u^{-2/3} = a \Rightarrow u = a^{-\frac{3}{2}}$

(a)에 의해 $\|y_\tau\|_{\mathcal{L}_\infty} \leq \left(\|u_\tau\|_{\mathcal{L}_\infty}\right)^{1/3}$

$\|u_\tau\|_{\mathcal{L}_\infty} \leq a^{-3/2}$ 에서는 $\|y_\tau\|_{\mathcal{L}_\infty} \leq \left(a^{-3/2}\right)^{1/3} = a^{-\frac{1}{2}} = (1/a)^{1/2} = \beta.$

$\|u_\tau\|_{\mathcal{L}_\infty} \geq a^{-3/2}$ 에서는 $\|y_\tau\|_{\mathcal{L}_\infty} \leq a\|u_\tau\|_{\mathcal{L}_\infty}$

For positive a, 위 부등호의 우변은 모두 양수.

$\therefore$ 모든 $\|u_\tau\|_{\mathcal{L}_\infty}$ 에 대해

$$\|y_\tau\|_{\mathcal{L}_\infty} \leq a\|u_\tau\|_{\mathcal{L}_\infty} + \underbrace{\left(\frac{1}{a}\right)^{1/2}}_{=\beta} \quad //$$

#5.11 (3) $\dot{x}_1 = (x_1 + u)(\|x\|_2^2 - 1)$

$\dot{x}_2 = x_2(\|x\|_2^2 - 1)$

$y = x_1$

Sol)

$V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$

$u=0$ 일때

$$\dot{V} = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1(x_1 + \cancel{u})(\|x\|_2^2 - 1) + x_2^2(\|x\|_2^2 - 1)$$

$$= \|x\|_2^2(\|x\|_2^2 - 1)$$

$\|x\|_2^2 > 1$ 에서 출발하는 solution은 발산 $\Rightarrow$ output y is not bounded

$\Rightarrow$ not L_∞ stable

For $\|x\|_2^2 \le 1 - 2\varepsilon$, $\dot{V} \le -\varepsilon\|x\|_2^2$, $\varepsilon > 0$ (식 (5.7)에 해당)

$\therefore$ Theorem 5.1의 조건을 만족한다.

$$\begin{cases} c_1 = c_2 = \frac{1}{2} \\ c_3 = \varepsilon \\ c_4 = 1 \\ L = 1 \\ \eta_1 = 1 \\ \eta_2 = 0 \end{cases}$$

$\therefore$ small-signal finite gain L_∞ stable 하다.

#5.18

$$\dot{x} = f(x) + G(x)u + K(x)\omega$$

$$y = h(x)$$

$$f(0) = 0 \ . \ h(0) = 0.$$

$$u = -G^T(x)\left(\frac{\partial V}{\partial x}\right)^T$$

$$\frac{\partial V}{\partial x} f(x) + \frac{1}{2}\frac{\partial V}{\partial x}\left[\frac{1}{\gamma^2}K(x)K^T(x) - G(x)G^T(x)\right]\left(\frac{\partial V}{\partial x}\right)^T + \frac{1}{2}h^T(x)h(x) \le 0$$

sol)

$$\dot{x} = f(x) - G(x)G^T(x)\left(\frac{\partial V}{\partial x}\right)^T + K(x)\omega$$

$$\omega \to \begin{bmatrix} y \\ u \end{bmatrix} \qquad \dot{x} = f'(x) + K(x)\omega$$

Theorem 5.5처럼 해보면, $y' = \begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} h(x) \\ -G^T(x)\left(\frac{\partial V}{\partial x}\right)^T \end{bmatrix} = h'(x)$

$$\frac{\partial V}{\partial x} f'(x) + \frac{\partial V}{\partial x} K(x)\omega = -\frac{1}{2}\gamma^2\left\|\omega - \frac{1}{\gamma^2}K^T(x)\left(\frac{\partial V}{\partial x}\right)^T\right\|_2^2 + \frac{\partial V}{\partial x}f'(x)$$

$$+\frac{1}{2\gamma^2}\frac{\partial V}{\partial x}K(x)K^T(x)\left(\frac{\partial V}{\partial x}\right)^T + \frac{1}{2}\gamma^2\|\omega\|_2^2$$

$f'(x) = f(x) - G(x)G^T(x)\left(\frac{\partial V}{\partial x}\right)^T$ 이므로 위 부등식을 이용

$$\frac{\partial V}{\partial x} f'(x) + \frac{\partial V}{\partial x}K(x)\omega \le \frac{1}{2}\gamma^2\|\omega\|_2^2 - \frac{1}{2}\|y'\|_2^2 - \frac{1}{2}\gamma^2\left\|\omega - \frac{1}{\gamma^2}K^T(x)\left(\frac{\partial V}{\partial x}\right)^T\right\|_2^2$$

$$\le \frac{1}{2}\gamma^2\|\omega\|_2^2 - \frac{1}{2}\|y'\|_2^2$$

$$V(x(\tau)) - V(x_0) \le \frac{1}{2}\gamma^2\int_0^\tau \|\omega\|_2^2\,dt - \frac{1}{2}\int_0^\tau \|y'(t)\|_2^2\,dt$$

Using $V(x) \ge 0$

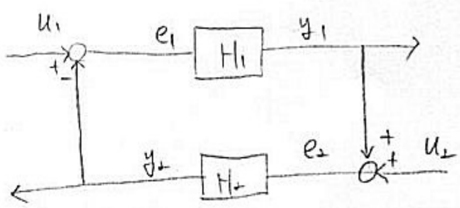
$$\int_0^\tau \|y'(t)\|_2^2\,dt \le \gamma^2\int_0^\tau \|\omega(t)\|_2^2\,dt + 2V(x_0)$$

$\sqrt{a^2+b^2} \le a+b$ for nonnegative number , square root

$$\|y'_\tau\|_{L_2} \le \gamma\|u_\tau\|_{L_2} + \sqrt{2V(x_0)}$$

$\therefore$ finite gain L_2 stable with L_2 gain less than or equal to γ. //

#5.21



$(u_1, u_2) \to (y_1, y_2) \Leftrightarrow (u_1, u_2) \to (e_1, e_2)$

* if($\Leftarrow$)조건과 only if($\Rightarrow$)조건을 증명하기 위해서 먼저 한 조건을 가정한 다음에 그 가정을 이용해서 결론을 이끌어 내야 함.

To show that $A \Rightarrow B$, assume A holds.
By using A, we conclude that B holds.

$$e_{1\tau} = u_{1\tau} - (H_2 e_2)_\tau$$
$$e_{2\tau} = u_{2\tau} + (H_1 e_1)_\tau$$

By theorem 5.6 의 proof

"only if" $\Rightarrow$

$$\|e_{1\tau}\|_{\mathcal{L}} \le \frac{1}{1-\gamma_1\gamma_2}\left(\|u_{1\tau}\|_{\mathcal{L}} + \gamma_2\|u_{2\tau}\|_{\mathcal{L}} + \beta_2 + \gamma_2\beta_1\right) \quad (5.40)$$

$$\|e_{2\tau}\|_{\mathcal{L}} \le \frac{1}{1-\gamma_1\gamma_2}\left(\|u_{2\tau}\|_{\mathcal{L}} + \gamma_1\|u_{1\tau}\|_{\mathcal{L}} + \beta_1 + \gamma_1\beta_2\right) \quad (5.41)$$

only if "$\Leftarrow$"

$$\begin{aligned}
\|y_{1\tau}\|_{\mathcal{L}} &\le \gamma_1\|e_{1\tau}\|_{\mathcal{L}} + \beta_1 \\
&\le \gamma_1\left[\frac{1}{1-\gamma_1\gamma_2}\left(\|u_{1\tau}\|_{\mathcal{L}} + \gamma_2\|u_{2\tau}\|_{\mathcal{L}} + \beta_2 + \gamma_2\beta_1\right)\right] + \beta_1 \\
&= \frac{\gamma_1}{1-\gamma_1\gamma_2}\|u_{1\tau}\|_{\mathcal{L}} + \frac{\gamma_1\gamma_2}{1-\gamma_1\gamma_2}\|u_{2\tau}\|_{\mathcal{L}} + \frac{\gamma_1(\beta_2+\gamma_2\beta_1)}{1-\gamma_1\gamma_2} + \beta_1
\end{aligned}$$

$$\|y_{2\tau}\|_{\mathcal{L}} \le \frac{\gamma_2}{1-\gamma_1\gamma_2}\|u_{2\tau}\|_{\mathcal{L}} + \frac{\gamma_1\gamma_2}{1-\gamma_1\gamma_2}\|u_{1\tau}\|_{\mathcal{L}} + \frac{\gamma_2(\beta_1+\gamma_1\beta_2)}{1-\gamma_1\gamma_2} + \beta_2$$

mapping from $(u_1\ u_2)$ to (e_1, e_2) is finite gain $\mathcal{L}$ stable

증명 "$\Rightarrow$" from $(u_1\ u_2)$ to $(y_1\ y_2)$ is also finite gain $\mathcal{L}$ stable //

증명 "$\Leftarrow$" 마찬가지로 하면

$$e_{1\tau} = u_{1\tau} - y_2$$

$$\begin{aligned}
\|e_{1\tau}\|_{\mathcal{L}} &\le \|u_{1\tau}\|_{\mathcal{L}} + \|y_{2\tau}\|_{\mathcal{L}} \\
&\le \|u_{1\tau}\|_{\mathcal{L}} + \frac{\gamma_2}{1-\gamma_1\gamma_2}\|u_{2\tau}\|_{\mathcal{L}} + \frac{\gamma_1\gamma_2}{1-\gamma_1\gamma_2}\|u_{1\tau}\|_{\mathcal{L}} + \frac{\gamma_2(\beta_1+\gamma_1\beta_2)}{1-\gamma_1\gamma_2} + \beta_2 \\
&= \frac{1}{1-\gamma_1\gamma_2}\left(\|u_{1\tau}\|_{\mathcal{L}} + \gamma_2\|u_{2\tau}\|_{\mathcal{L}} + \beta_2 + \gamma_2(\beta_1 + \cancel{\gamma_1\beta_2}) + \beta_2 - \cancel{\beta_2\gamma_1\gamma_2}\right) \\
&= \frac{1}{1-\gamma_1\gamma_2}\left(\|u_{1\tau}\|_{\mathcal{L}} + \gamma_2\|u_{2\tau}\|_{\mathcal{L}} + \beta_2 + \beta_2 + \gamma_2\beta_1\right)
\end{aligned}$$

$$\|e_{2\tau}\|_{\mathcal{L}} \le \|u_{2\tau}\| + \|y_{1\tau}\|_{\mathcal{L}} \le \frac{1}{1-\gamma_1\gamma_2}\left(\|u_{2\tau}\|_{\mathcal{L}} + \gamma_1\|u_{1\tau}\|_{\mathcal{L}} + \beta_1 + \gamma_1\beta_2\right) //$$

5.3) (a) $y = u^{\frac{1}{3}}$ 이므로

$|y| \le |u|^{\frac{1}{3}}$

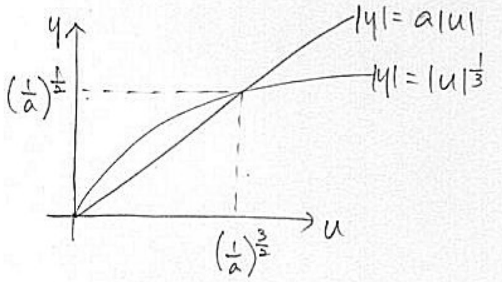
$\therefore \|y_\tau\|_{\mathcal{L}_\infty} \le (\|u_\tau\|_{\mathcal{L}_\infty})^{\frac{1}{3}}$

$\alpha(r) = r^{\frac{1}{3}}$ 이라 하자. $\alpha(\cdot)$는 class K_∞ function이다.

$\therefore \|y_\tau\|_{\mathcal{L}_\infty} \le \alpha(\|u_\tau\|_{\mathcal{L}_\infty})$

$\therefore$ 이 시스템은 $\mathcal{L}_\infty$ stable 이고 $\beta = 0$이므로 zero bias.

(b) $|y| = |u|^{\frac{1}{3}}$ 과 $|y| = a|u|$의 교차점은 $\left((\frac{1}{a})^{\frac{3}{2}}, (\frac{1}{a})^{\frac{1}{2}} \right)$이다.



따라서
$$\begin{cases} |u| \le (\frac{1}{a})^{\frac{3}{2}} \text{ 인 영역에서 } |y| \le (\frac{1}{a})^{\frac{1}{2}} \\ |u| > (\frac{1}{a})^{\frac{3}{2}} \text{ 인 영역에서 } |y| \le a|u| \end{cases}$$

$\therefore$ 모든 $|u| \ge 0$ 영역에서

$|y| \le (\frac{1}{a})^{\frac{1}{2}} + a|u|$,

$\therefore \gamma = a$, $\beta = (\frac{1}{a})^{\frac{1}{2}}$ 로 잡으면

$\|y_\tau\|_{\mathcal{L}_\infty} \le \gamma \|u_\tau\|_{\mathcal{L}_\infty} + \beta$

꼴이 되어 이 시스템이 finite-gain $\mathcal{L}_\infty$ stable임을 알 수 있다.

(c) ???

5. (1)(3)

$$\begin{cases} \dot{x}_1 = (x_1 + u)(\|x\|_2^2 - 1) \\ \dot{x}_2 = x_2(\|x\|_2^2 - 1) \\ y = x_1 \end{cases}$$

$V(x) = \frac{1}{2}(x_1^2 + x_2^2)$ 이라하면,

$$\dot{V}(x) = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1(x_1+u)(\|x\|_2^2 - 1) + x_2^2(\|x\|^2 - 1)$$

$$= (\|x\|^2 - 1)(x_1^2 + x_2^2 + x_1 u)$$

$$\leq (\|x\|^2 - 1)(\|x\|_2^2 + \|x\|_2 |u|)$$

If $u=0$,
$\dot{V} = -\|x\|_2^2(1 - \|x\|_2^2)$.

$$\leq -\|x\|_2^2 + \|x\|_2^4 + \|x\|_2^3 |u|$$

$\forall x$ s.t. $\|x\|_2 > 1$, $\dot{V} > 0$.

∴ not $\mathcal{L}_\infty$ stable..

$$= -(1-\theta)\|x\|_2^2 - \theta\|x\|_2^2 + \|x\|_2^4 + \|x\|_2^3 |u|, \quad 0 < \theta < 1$$

$\rho(s)$를 $-\theta y^2 + y^4 + y^3 s = 0$, $s \geq 0$. 의 가장 큰 실근이라고하면,

?

$\dot{V} < -(1-\theta)\|x\|_2^2$, $\underline{\forall \|x\|_2 \geq \rho(|u|)}$

$\rightarrow \|x\|_2 \leq \frac{-|u| + \sqrt{|u|^2 + 4\theta}}{2}$

∴ 이 시스템은 input-to-state stable.

$\forall x$ s.t. $\|x\|_2 < 1$

$\dot{x} = \frac{\partial f}{\partial x}\Big|_{x=0} x$; exp. stable

∴ small-signal finite-gain $\mathcal{L}_\infty$ stable..

$y = h(x) = x_1$ 이므로

$$\|h(t, x, u)\| \leq \alpha_1(\|x\|) + \alpha_2(\|u\|) + \eta$$

$\alpha_1(r) = r$, $\alpha_2(\cdot) = 0$, $\eta = 0$ 일때 위식 만족.

∴ 이 시스템은 $\mathcal{L}_\infty$ stable. (x)

5.18)

$$\begin{cases} \dot{x} = f(x) + G(x)u + K(x)w & \cdots ① \\ y = h(x) & \cdots ② \\ u = -G^T(x)\left(\frac{\partial V}{\partial x}\right)^T & \cdots ③ \end{cases}$$

closed-loop map from w to $\begin{bmatrix} y \\ u \end{bmatrix}$ 는

$$\begin{cases} \dot{x} = f_c(x) + G_c(x)w \\ y_c = h_c(x) \end{cases} \quad , \quad \begin{aligned} & f_c(x) = f(x) - G(x)G^T(x)\left(\frac{\partial V}{\partial x}\right)^T \\ & G_c(x) = K(x) \\ & y_c = \begin{bmatrix} y \\ u \end{bmatrix}, \quad h_c(x) = \begin{bmatrix} h(x) \\ -G^T(x)\left(\frac{\partial V}{\partial x}\right)^T \end{bmatrix} \end{aligned}$$

문제에서 주어진 조건들이 theorem 5.5 의 조건에 부합하므로

Hamilton-Jacobi inequality가 성립하면 finite-gain $\mathcal{L}_2$ stable이고

$\mathcal{L}_2$ gain이 γ 이하임을 알 수 있다.

$$\mathcal{H}(V, f_c, G, h, \gamma) = \frac{\partial V}{\partial x} f_c(x) + \frac{1}{2\gamma^2} \cdot \frac{\partial V}{\partial x} G_c(x) G_c^T(x) \left(\frac{\partial V}{\partial x}\right)^T + \frac{1}{2} h_c^T(x) h_c(x)$$

$$= \frac{\partial V}{\partial x}\left(f(x) - G(x)G^T(x)\left(\frac{\partial V}{\partial x}\right)^T\right) + \frac{1}{2\gamma^2} \cdot \frac{\partial V}{\partial x} K(x) K^T(x) \left(\frac{\partial V}{\partial x}\right)^T$$
$$+ \frac{1}{2} h^T(x) h(x) + \frac{1}{2}\left(\frac{\partial V}{\partial x}\right) G(x) G^T(x) \left(\frac{\partial V}{\partial x}\right)^T$$

$$\leq \frac{\partial V}{\partial x} f(x) + \frac{1}{2} \cdot \frac{\partial V}{\partial x}\left[\frac{1}{\gamma^2} K(x) K^T(x) - G(x) G^T(x)\right]\left(\frac{\partial V}{\partial x}\right)^T + \frac{1}{2} h_c^T(x) h_c(x) \cdots ⑤$$

⑤ 식은 문제에서 주어진 조건에 따라 ⑤ ≤ 0

$\therefore$ $\mathcal{H}(V, f, G, h, \gamma) \leq 0$ 이므로 closed-loop map

from w to $\begin{bmatrix} y \\ u \end{bmatrix}$ 는 finite-gain $\mathcal{L}_2$ stable 이고

$\mathcal{L}_2$ gain은 γ 이하이다.

5.21) (u_1, u_2)에서 (y_1, y_2)로의 mapping이 finite $\mathcal{L}$ stable.

$\Leftrightarrow$ (u_1, u_2)에서 (e_1, e_2)로의 mapping이 finite $\mathcal{L}$ stable) 증명.

i) (u_1, u_2)에서 (y_1, y_2)로의 mapping이 finite $\mathcal{L}$ stable

$\Rightarrow$ (u_1, u_2)에서 (e_1, e_2)로의 mapping이 finite $\mathcal{L}$ stable

(u_1, u_2)에서 (y_1, y_2)로의 mapping이 finite $\mathcal{L}$ stable이면,

$$\begin{cases} \|y_{1\tau}\|_{\mathcal{L}} \le r_1 \|u_{1\tau}\|_{\mathcal{L}} + \beta_1 & \cdots \text{①} \\ \|y_{2\tau}\|_{\mathcal{L}} \le r_2 \|u_{2\tau}\|_{\mathcal{L}} + \beta_2 & \cdots \text{②} \end{cases}$$

figure 5.1 에서
$$\begin{cases} e_1 = u_1 - y_2 & \cdots \text{③} \\ e_2 = u_2 + y_1 & \cdots \text{④} \end{cases}$$

①식에 ④을 대입하면

$$\|y_{1\tau}\|_{\mathcal{L}} = \|e_{2\tau} - u_{2\tau}\|_{\mathcal{L}} \ge \|e_{2\tau}\|_{\mathcal{L}} - \|u_{2\tau}\|_{\mathcal{L}}$$

$$\|e_{2\tau}\|_{\mathcal{L}} - \|u_{2\tau}\|_{\mathcal{L}} \le \|y_{1\tau}\|_{\mathcal{L}} \le r_1 \|u_{1\tau}\|_{\mathcal{L}} + \beta_1$$

$$\therefore \|e_{2\tau}\|_{\mathcal{L}} \le r_1 \|u_{1\tau}\|_{\mathcal{L}} + \|u_{2\tau}\|_{\mathcal{L}} + \beta_1 \quad \cdots \text{⑤}$$

②식에 ③은 대입하면,

$$\|y_{2\tau}\|_{\mathcal{L}} = \|u_{1\tau} - e_{1\tau}\|_{\mathcal{L}} = \|e_{1\tau} - u_{1\tau}\|_{\mathcal{L}} \ge \|e_{1\tau}\|_{\mathcal{L}} - \|u_{1\tau}\|_{\mathcal{L}}$$

$$\|e_{1\tau}\|_{\mathcal{L}} - \|u_{1\tau}\|_{\mathcal{L}} \le \|y_{2\tau}\|_{\mathcal{L}} \le r_2 \|u_{2\tau}\|_{\mathcal{L}} + \beta_2$$

$$\therefore \|e_{1\tau}\|_{\mathcal{L}} \le \|u_{1\tau}\|_{\mathcal{L}} + r_2 \|u_{2\tau}\|_{\mathcal{L}} + \beta_2 \quad \cdots \text{⑥}$$

$\therefore$ 식⑤, ⑥에 의해서 (u_1, u_2)에서 (e_1, e_2)로의 mapping이 finite gain $\mathcal{L}$ stable 임을 알 수 있다.

ii) (u_1, u_2)에서 (e_1, e_2)로의 mapping이 finite $\mathcal{L}$ stable

$\Rightarrow$ (u_1, u_2)에서 (y_1, y_2)로의 mapping이 finite $\mathcal{L}$ stable

(u_1, u_2)에서 (e_1, e_2)로의 mapping이 finite $\mathcal{L}$ stable이면,

$$\begin{cases} \|e_{1\tau}\|_{\mathcal{L}} \le \gamma_1 \|u_{1\tau}\|_{\mathcal{L}} + \beta_1 & \cdots ⑦ \\ \|e_{2\tau}\|_{\mathcal{L}} \le \gamma_2 \|u_{2\tau}\|_{\mathcal{L}} + \beta_2 & \cdots ⑧ \end{cases}$$

③식에 의해서

$$\|e_{1\tau}\|_{\mathcal{L}} = \|u_{1\tau} - y_{2\tau}\|_{\mathcal{L}} = \|y_{2\tau} - u_{1\tau}\|_{\mathcal{L}} \ge \|y_{2\tau}\|_{\mathcal{L}} - \|u_{1\tau}\|_{\mathcal{L}}$$

⑦식에서

$$\|y_{2\tau}\|_{\mathcal{L}} - \|u_{1\tau}\|_{\mathcal{L}} \le \|e_{1\tau}\|_{\mathcal{L}} \le \gamma_1 \|u_{1\tau}\|_{\mathcal{L}} + \beta_1$$

$$\therefore \|y_{2\tau}\|_{\mathcal{L}} \le (\gamma_1 + 1) \|u_{1\tau}\|_{\mathcal{L}} + \beta_1 \quad \cdots ⑨$$

④식에 의해서

$$\|e_{2\tau}\|_{\mathcal{L}} = \|y_{1\tau} + u_{2\tau}\|_{\mathcal{L}} \ge \|y_{1\tau}\|_{\mathcal{L}} - \|u_{2\tau}\|_{\mathcal{L}}$$

⑧식에서

$$\|y_{1\tau}\|_{\mathcal{L}} - \|u_{2\tau}\|_{\mathcal{L}} \le \|e_{2\tau}\|_{\mathcal{L}} \le \gamma_2 \|u_{2\tau}\|_{\mathcal{L}} + \beta_2$$

$$\therefore \|y_{1\tau}\|_{\mathcal{L}} \le (\gamma_2 + 1) \|u_{2\tau}\|_{1} + \beta_2 \quad \cdots ⑩$$

$\therefore$ 식 ⑨, ⑩에 의해서 (u_1, u_2)에서 (y_1, y_2)로의 mapping은 finite gain $\mathcal{L}$ stable 이다.

$\Rightarrow$ i), ii) 에 의해서 필요충분조건 증명.