

물리화학 Z homework #1 solution

1. (1)  $c(x, t) = \frac{n_0}{A(\pi D t)^{1/2}} e^{-x^2/4Dt}$

$$\langle x^2 \rangle^{1/2} = \int_0^\infty \frac{1}{(\pi D t)^{1/2}} x^2 e^{-x^2/4Dt} dx$$

let  $y = \frac{x^2}{4Dt}$   $dy = 2x \frac{dx}{4Dt}$   $x = \sqrt{4Dt} \sqrt{y}$

$$\langle x^2 \rangle = \int_0^\infty \frac{1}{(\pi D t)^{1/2}} (4Dt \cdot y) \frac{2Dt}{\sqrt{4Dt} \sqrt{y}} e^{-y} dy$$

$$= \frac{4Dt}{\sqrt{\pi}} \int_0^\infty y^{1/2} e^{-y} dy$$

$$= \frac{4Dt}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}\right)$$

$$= \frac{4Dt}{\sqrt{\pi}} \frac{1}{2} \left(-\frac{1}{2}\right)! = \frac{4Dt}{\sqrt{\pi}} \frac{1}{2} \cdot \frac{1}{2} \sqrt{\pi} = 2Dt$$

$$\langle x^2 \rangle^{1/2} = \sqrt{2Dt}$$

(2)  $\langle x^2 \rangle^{1/2} = \sqrt{2Dt} = \sqrt{2(4.67 \times 10^{-10} \text{m}^2/\text{s})(60\text{s})} = 2.36 \times 10^{-4} \text{m}$ ,

(3)  $\langle x^2 \rangle^{1/2} = \sqrt{2\alpha t} = \sqrt{2 \cdot \frac{k}{\rho C_p} \cdot t} = \sqrt{2 \cdot \frac{10^5}{3.4} \cdot 60} = 0.01 \text{m}$ ,

Z. (1) In gases the frequency of collisions increases with increasing temperature; hence the viscosity of gases increases with temperature since the molecules are very few apart from each other.

In liquids, the molecules are very close to each other, leading to the strong forces of attraction between them. As the temperature increases, more and more molecules are likely to have sufficient kinetic energy to overcome the force of attraction resulting decreased viscosity

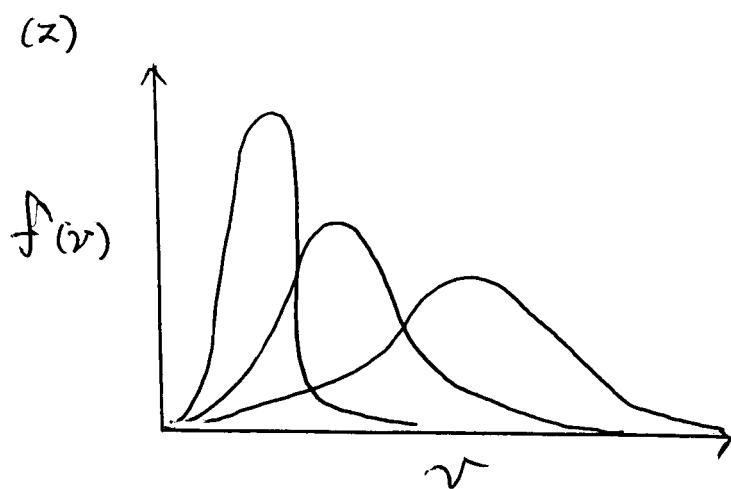
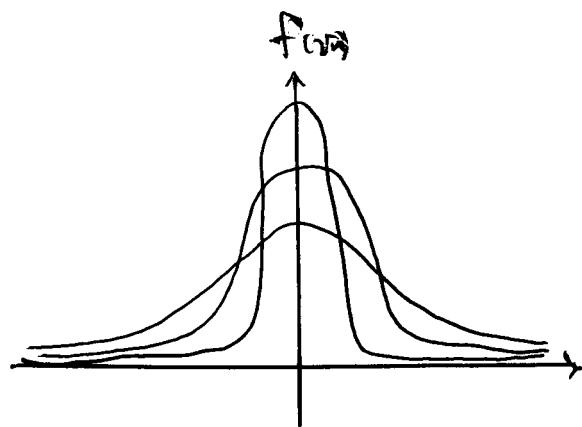
(2) fraction of molecules in the speed range from 200 to 250 ms<sup>-1</sup> is

$$F = \int_{200}^{250} f(v) dv$$

$$\begin{aligned} & \int_{200}^{250} 4\pi \left( \frac{M}{2\pi RT} \right)^{\frac{3}{2}} v^2 \exp\left(-\frac{Mv^2}{2RT}\right) dv \\ & 4\pi \left( \frac{M}{2\pi RT} \right)^{\frac{3}{2}} v^2 \exp\left(-\frac{Mv^2}{2RT}\right) dV \\ & 4\pi \left( \frac{44.0 \times 10^{-3} k_J / \text{mol}}{2\pi (8.3145 \text{ J/K.mol})(300 \text{ K})} \right)^{\frac{3}{2}} (225 \text{ m/s})^2 \\ & \times \exp\left(\frac{-44.0 \times 225^2}{2 \times 8.3145 \times 300}\right) (50 \text{ m/s}) \\ & = 9.6 \times 10^{-2} \end{aligned}$$

3.

(a)



$$\begin{aligned}
 (3) \quad K &= \Lambda_m C = (\nu_+ \lambda_+ + \nu_- \lambda_-) C \\
 &= (\nu_+ Z_+ U_+ F + \nu_- Z_- U_- F) C
 \end{aligned}$$

$$\frac{K}{F(U_{H^+} + U_{OH^-})} = \frac{5.5 \times 10^{-6} / \text{J} \cdot \text{m}}{(96485 \text{ C/mol})(5.689 \times 10^{-7} \text{ m}^2/\text{V} \cdot \text{s})}$$

$$= 1 \times 10^{-4} \text{ mol/cm}^3 = 1 \times 10^{-7} \text{ mol/L}$$

$$Kw = (1 \times 10^{-7})^2 = 1 \times 10^{-14} \text{ (at } 25^\circ\text{C})$$

4.

$$(1) D = \frac{uRT}{zF} \quad u = \frac{DzF}{RT} = \frac{(1.09 \times 10^{-9} m^2/s)(1)(96450 C/mol)}{(83145 J/K\cdot mol)(298 K)}$$

$$= 4.24 \times 10^{-8} m^2/s \cdot V$$

$$(2) \langle x^2 \rangle = 2Dt$$

$$r^2 = x^2 + y^2 + z^2 \quad \langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle = 3 \langle x^2 \rangle \\ = 6Dt$$

$$t = \frac{\langle r^2 \rangle}{6D} = \frac{(1 \times 10^{-2} m)^2}{(6)(1.09 \times 10^{-9} m^2/s)} = 1.4 \times 10^{4.5}$$

$$(3) D = \frac{kT}{6\pi\eta a} \Rightarrow a = \frac{kT}{6\pi\eta D} = \frac{(1.38 \times 10^{-23} J/K)(298 K)}{6\pi(1 \times 10^{-3} kg/m\cdot s)(1.09 \times 10^{-9} m^2/s)}$$

$$a = 2.00 \times 10^{-10} = 200 \text{ pm} \quad //$$

5. Text book, 185 page,,