Home Problem Set #1 Due : September 10 (Wed), 2008

1. Answer the following questions related to the wave-particle duality.

(a) When an electron (mass *m*) is moving with the velocity of υ, what is the wave length of electron wave suggested by de Broglie?

(b) Find the de Broglie wavelength of a He atom having thermal energy at T = 300 K.

(c) Represent the linear momentum *p* and kinetic energy *E* of the electron with wave number *k* , mass *m*, and Plank constant *h*.

2. (a) What should be the energy of an electron so that the associated electron waves have a wavelength of 600nm?

(b) Find the phase and group velocities of this de Broglie wave in (a).

(c) What is the energy of a light quantum (photon) which has a wavelength of 600nm? Compare the energy with the electron wave energy calculated in (a) and discuss the difference.

3. If one were to ask, “What is the location of an electron if we know that it is moving with a specific velocity?”, would this be a meaningful question? Would you prefer to believe that electrons actually have specific positions and velocities simultaneously but the world is such that we can’t know them, or that electrons are entities such that thinking in terms of simultaneous position and velocity is an inappropriate thing to do?

4. Derive $\Psi_1 + \Psi_2 = \Psi = 2\cos(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x)\cdot\sin[(k+\frac{\Delta k}{2})x - (\omega + \frac{\Delta\omega}{2})t]$

by adding $\Psi_1 = \sin[kx - \omega t]$ and $\Psi_2 = \sin[(k+\Delta k)x - (\omega+\Delta\omega)t]$

5. Derive $\lambda p = h$, by combining

$$\begin{aligned}&\textit{Momentum}: p = m\upsilon \\ &\textit{Speed}\ \text{of light}: c = \nu\lambda \\ &\text{Einstein's mass-energy equivalence}: E = mc^2\end{aligned}$$

6. Let us assume w is the velocity of propagation for a de Broglie wave of a particle of momentum *p* and total energy E. Then we obtain $w = \lambda\nu = (h/p)(E/h) = E/p$

Assuming the particle is moving at nonrelativistic velocity, we find $w = E/p = (m\upsilon^2/2)/m\upsilon = \upsilon/2$. This result means the propagation velocity of the de Broglie wave of the particle is only half of the particle velocity and thus seems disturbing because the matter wave should keep up with the particle. What is wrong for this derivation?

Home Problem Set #2 Due : September 29, 2008

1. Describe the energy for
 (a) a free electron
 (b) a strongly bound electron
 (c) an electron in a periodic potential
 Why do we get these different band schemes?

2. State the two Schrödinger equations for electrons in a periodic potential field (Kronig-Penny model). Use for their solutions, instead of the Bloch function, the trial solution

$$\Psi(x) = A e^{ikx}$$

Discuss the result. (*Hint*: For free electrons $V_o = 0$)

3. The differential equation for an undamped vibration is $a\frac{d^2u}{dx^2} + bu = 0$ (1)
 whose solution is $u = Ae^{ikx} + Be^{-ikx}$ (2)
 Where $k = \sqrt{b/a}$
 Prove that (2) is indeed a solution of (1)

4. Derive $E = \frac{me^4}{2(4\pi\varepsilon_0\hbar)^2}\frac{1}{n^2} = -13.6\cdot\frac{1}{n^2}(eV) \quad (4.18a)$

in a semiclassical way by assuming that the centripetal force of an electron, mv^2/r, is counterbalanced by the Coulombic attraction force, $-e^2/4\pi\varepsilon_0 r^2$, between the nucleus and the orbiting electron. Use Bohr's postulate which states that the angular momentum $L = mvr$ (v = linear electron velocity and r = radius of the orbiting electron) is a multiple integer of Planck's constant (i.e., nh). (*Hint*: The kinetic energy of the electron is $E = mv^2/2$)

5. Consider an electron trapped in a one-dimensional box with infinitely high potential. The box extends from $x = 0$ to $x = a$. The electron has the mass m and the total energy E.

(a) Find the energy eigenvalue of the electron by solving Schrödinger equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

(b) If the lowest energy possible for the electron is 10.00 eV, what are the two higher energies the particle can have?

(c) Find the eigenfunction ψ_n corresponding to E_n.

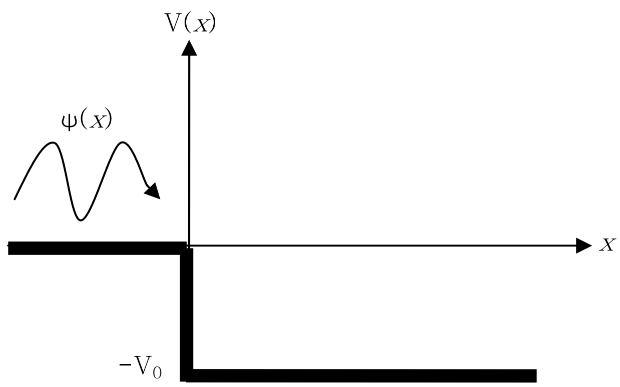
(d) Find the probability density that the electron can be found between $x = 0$ and $a/3$ for the second excited sate (n =3).

6. An electron of mass m and kinetic energy $\boldsymbol{E > 0}$ approaches a potential drop V_0

(a) Write down the equations of two regions ($x \geq 0$, $x \leq 0$) from time-independent Schrödinger equation.

(b) Assume there is no incoming wave to the right, so you can get three amplitude constants. Let A : incident amplitude, B : reflected amplitude, C : transmitted amplitude.

Determine the reflection coefficient R. ($R = \dfrac{|B|^2}{|A|^2}$)



(c) What is R if $E = V_0 / 3$?

Calculate the transmission coefficient T in this case from R + T = 1.

Home Problem Set #3 Due : October 6 (Mon), 2008

1. Explain the difference in allowed energy levels between bound electrons, free electrons, and electrons in crystals.

2. Calculate how much the kinetic energy of a free electron at the corner of the first Brillouin zone of a simple cubic lattice (three dimensions!) is larger than that of an electron at the midpoint of the face.

3. Construct the first four Brillouin zones for a simple cubic lattice in two dimensions.

4. Calculate the shape of the free electron bands for the cubic primitive crystal structure for $n = 1$ and $n = -2$ (See Fig. 5.6).

5. Calculate the main lattice vectors in reciprocal space of an fcc crystal.

6. If $b_1 \cdot t_1 = 1$ is given (see equation (5.14)), does this mean that b_1 is parallel to t_1?

Home Problem Set #4 Due : October 13 (Mon), 2008

1. Are there more electrons on the bottom or in the middle of the valence band of a metal? Explain.

2. At what temperature can we expect a 10% probability that electrons in silver have an energy which is 1% above the Fermi energy? (E_F = 5.5 eV)

3. Calculate the density of states of 1 m^3 of copper at the Fermi level ($m^* = m_0$, E_F = 7 eV). *Note* : Take 1eV as energy interval. (Why?)

4. The density of states at the Fermi level (7 eV) was calculated for 1cm^3 of a certain metal to be about 10^{21} energy states per electron volt. Someone is asked to calculate the number of electrons for this metal using the Fermi energy as the maximum kinetic energy which the electrons have. He argues that because of the Pauli principle, each energy state is occupied by two electrons. Consequently, there are 2 x 10^{21} electrons in that band.
   (a) What is wrong with that argument?
   (b) Why is the answer, after all, not too far from the correct numerical value?

5. (a) Calculate the number of free electrons per cubic centimeter in copper, assuming that the maximum energy of these electrons equals the Fermi energy ($m^* = m_0$).
   (b) How does this result compare with that determined directly from the density and the atomic mass of copper? Hint : Consider equation (7.5)
   (c) How can we correct for the discrepancy?
   (d) Does using the effective mass decrease the discrepancy?

6. We stated in the text that the Fermi distribution function can be approximated by classical Boltzmann statistics if the exponential factor in the Fermi distribution function is significantly larger than one.
   (a) Calculate $E - E_F = nk_BT$ for various values of n and state at which value for n,

$$\exp\left(\frac{E - E_F}{k_B T}\right)$$

   can be considered to be "significantly larger" than 1 (assume T = 300 K).
   (b) For what *energy* can we use Boltzmann statistics? (Assume E_F = 5 eV and $E - E_F = 4k_BT$)

Home Problem Set #5 Due : October 20 (Mon), 2008

1. Calculate the number of free electrons for gold using its density and its atomic mass.

2. Answer the questions related to electrical conduction in metals and alloys.
   (a) Using the Drude postulation, derive the following equation for conductivity as a function of N_f (number of free electron per unit volume), electron charge e, electron mass m, and relaxation time τ ;

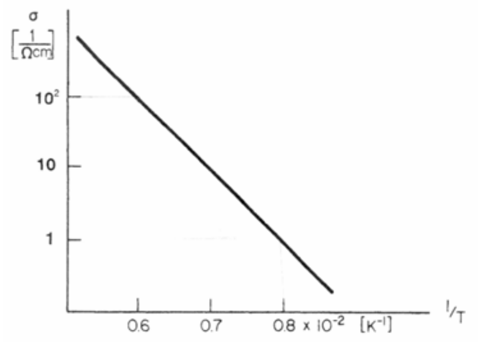
$$\sigma = \frac{N_f e^2 \tau}{m}$$

   (b) Explain briefly why the above Drude classical conduction theory needs a modification, and also explain briefly how it is modified by quantum mechanical consideration.
   (c) Describe the temperature dependence of resistivity in a pure metal. Explain the reason for this behavior on the basis of the equation in (a).

3. Calculate the number of electrons in the conduction band for silicon at T = 300 K. (Assume $m^*_e / m_0 = 1$.)

4. Calculate the Fermi energy of an intrinsic semiconductor at T $\neq$ 0 K. (*Hint* : Give a mathematical expression for the fact that the probability of finding an electron at the top of the valence band plus the probability of finding an electron at the bottom of the conduction band must be 1.) Let $N_e \equiv N_p$ and $m^*_e \equiv m_h^*$.

5. In the figure below, σ is plotted as a function of the reciprocal temperature for an intrinsic semiconductor. Calculate the gap energy. (*Hint*: Use (8.14) and take the ln from the resulting equation.)



Home Problem Set #6 Due : November 17 (Mon), 2008

1. Consider a silicon crystal containing 10^{12} phosphorous atoms per cubic centimeter. Is the conductivity increasing or decreasing when the temperature is raised from 300° to 350°C ? Explain by giving numerical values for the mechanisms involved.

2. Consider a semiconductor with 10^{13} donors/cm^3 which has a binding energy of 10 meV.
   (a) What is the concentration of extrinsic conduction electrons at 300 K ?
   (b) Assuming a gap energy of 1 eV (and $m^* \equiv m_0$), what is the concentration of intrinsic conduction electrons?
   (c) Which contribution is larger ?

3. You are given a *p*-type doped silicon crystal and are asked to make an ohmic contact. What material would you use?

4. Describe the band diagram and function of a *p*-*n*-*p* transistor.

5. Calculate the room-temperature saturation current and the forward current at 0.3 V for a silver/*n*-doped silicon Schottky-type diode. Take for the active area 10^{-8} m^2 and $C = 10^{19}$ A/m^{2}K^2.

6. Describe the energy bands for a metal and a semiconductor *before* and *after* contact.
   a) *n*-type, $\phi_M > \phi_S$
   b) *n*-type, $\phi_M < \phi_S$
   c) *p*-type, $\phi_M > \phi_S$
   d) *p*-type, $\phi_M < \phi_S$

7. Calculate the mobility of the oxygen ions in UO_2 at 700 K. The diffusion coefficient of O^{2-} at this temperature is 10^{-13} cm^2/s. Compare this mobility with electron or hole mobilities in semiconductors (see Appendix 4). Discuss the difference! (*Hint*: O^{2-} has two charges!)

8. Calculate the activation energy for ionic conduction for a metal ion in an ionic crystal at 300 K. Take Do = 10^{-3} m^2/s and D = 10^{-17} m^2/s.

9. Show that $E = E_{vac}/ \varepsilon$ [Eq. (9.13)] by combining Eqs. (7.3), (9.9), and (9.11) and their equivalents for vacuum.

10. Show that the dielectric polarization is $P = (\varepsilon - 1) \varepsilon_0 E$. What values do P and D have for vacuum?

11. Derive the Einstein relation using Fick's first law.

Home Problem Set # 7 Due : December 8 (Mon), 2008

1. Represent reflectivity, *R* as a function of refractive index, *n* and damping constant, *k*. On the basis of this relationship, explain why insulators like ceramics and polymers have very low *R* values.

2. Explain the reason for very high reflectivity in the IR region.

3. Express *n* and *k* in terms of ε and σ (or ε_1 and ε_2) by using $\varepsilon = n^2 - k^2$ and $\sigma = 4\pi\varepsilon_0 nk\nu$. (Compare with (10.15) and (10.16).)

4. Calculate the reflectivity of silver and compare it with the reflectivity of flint glass (n = 1.59). Use λ = 0.6 μm

5. Derive the Hagen-Rubens relation from (10.29). (*Hint*: In the IR region $\varepsilon_2^2 >> \varepsilon_1^2$ can be used. Justify this approximation.)

6. The transmissivity of a piece of glass of thickness *d* = 1 cm was measured at λ = 589 nm to be 89 %. What would the transmissivity of this glass be if the thickness were reduced to 0.5 cm?

7. Calculate the reflectivity of sodium in the frequency ranges $\nu > \nu_1$ and $\nu < \nu_1$ using the theory for free electrons without damping. Sketch R versus frequency.

8. Calculate the reflectivity of gold at $\nu = 9 \times 10^{12}$ s^{-1} from its conductivity. Is the reflectivity increasing or decreasing at this frequency when the temperature is increased? Explain.

9. The optical properties of an absorbing medium can be characterized by various sets of parameters. One such set is the index of refraction and the damping constant. Explain the physical significance of those parameters, and indicate how they are related to the complex dielectric constant of medium. What other sets of parameters are commonly used to characterize the optical properties? Why are there always “sets” of parameters?

10. Derive the Drude equations from (11.45) and (11.46) by setting $\nu_0 \rightarrow 0$.

11. Below the reflection spectra for two materials A and B are given.

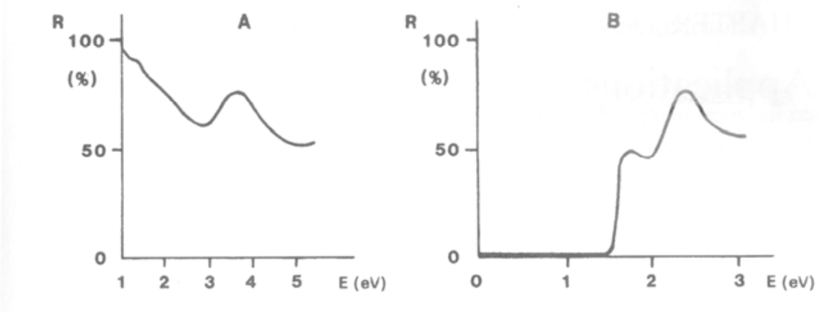
(a) What type of material belongs to reflection spectrum A, what type to B? (Justify). Note the scale difference!

(b) For which colors are these (bulk) materials transparent?

(c) What is the approximate threshold energy for interband transitions for these materials?

(d) For which of the materials would you expect intraband transitions in the infrared region? (Justify.)

(e) Why do these intraband transitions occur in this region?



12. How thick is the depletion layer for an electro-optical waveguide when the index of refraction (n_3 = 3.6) increases in Medium 2 by 0.1 %? Take n_1 = 1, λ_0 = 1.3 µm, and zeroth-order mode.

Home Problem Set # 8 Due : December 12 (Fri), 2008

1. An electromagnet is a helical winding of wire through which an electric current flows. Such a "solenoid" of 1000 turns is 10 cm long and is passed through by a current of 2A. What is the field strength in Oe and A/m?

2. Calculate the diamagnetic susceptibility of germanium. Take $\bar{r}$ = 0.92 Å. (*Note*: Check your units! Does χ come out unitless? Compare your result with that listed in Table 14.1.)

3. Estimate the number of Bohr magnetons for iron and cobalt ferrite from their electron configuration, as done in the text. Compare your results with those listed in Table 15.3. Explain the discrepancy between experiment and calculation. Give the chemical formula for these ferrites.

4. Compare the experimental saturation magnetization, M_{s0} (Table 15.1 third column), with the magnetic moment, μ_m, at 0 K for ferromagnetic metals (Table 16.1). What do you notice? Estimate the degree of *d*-band filling for iron and cobalt.

5. From the results obtained in the previous problem, calculate the number of Bohr magnetons for crystalline (solid) iron and cobalt and compare your results with those listed in Table 16.1. What is the number of Bohr magnetons for an iron *atom* and a cobalt atom? What is the number of Bohr magnetons for iron and cobalt ferrite?

6. Calculate C_v at high temperatures (500 K) by using the quantum mechanical equation derived by Einstein. Assume an Einstein temperature of 250 K, and convince yourself that C_v approaches the classical value at high temperatures.

7. Calculate the electronic specific heat for E_F = 5 eV and T = 300 K. How does your result compare with the experimental value of 25 (J/mol K)?