Homework Assignment 1

Issued: Sep 10 Due: Sep 24 (Wed)

1. Prove the following:

(a) $||A||_1 = \max_{1 \leq j \leq m} \sum_{i=1}^{n} |A_{ij}|$

(b) Eigenvectors that correspond to distinct eigenvalues are linearly independent.

(c) Given a matrix $A \in \mathbb{R}^{n \times n}$, there exists a matrix $Q \in \mathbb{C}^{n \times n}$ such that

$$Q^*Q = I, \quad \text{and} \quad Q^*AQ = \Lambda \text{ is upper triangular,}$$

and the eigenvalues of Q^*AQ are the same as the eigenvalues of A.

(d) Let λ be an eigenvalue of an orthogonal matrix. Show that $|\lambda| = 1$.

(e) For symmetric matrices,

$$\rho(A) = ||A||_2 \ .$$

(f) For any matrix norm,

$$\rho(A) \leq ||A|| \ .$$

2. Consider the system

$$G(s) = \frac{1}{s^2 + 2\zeta s + \omega_n^2} \qquad (\zeta > 0)$$

Determine $||G||_2$, and $||G||_\infty$.

3. A mass/spring/damper system can be described by a state-space representation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ z(t) &= Cx(t) \end{aligned}$$

where $u = [u_1; u_2]^T$ denote the forces acting on the masses, and $z = [z_1; z_2]^T$ are the position of the masses. The system matrices are given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/m_1 & k_1/m_1 & -b_1/m_1 & b_1/m_1 \\ -k_1/m_2 & -(k_1+k_2)/m_2 & -b_1/m_2 & -(b_1+b_2)/m_2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_1 & 0 \\ 0 & 1/m_2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The numerical values are $m_1 = 1; m_2 = 2, k_1 = 1; k_2 = 4, b_1 = 0.2; b_2 = 0.1$. Plot the singular values of the system as functions of frequency. Determine the H_∞ norm of the system directly from the singular value plot. Determine the sinusoidal input signal of the form $u(t) = [a_1 sin(\omega t + \alpha_1); a_2 sin(\omega t + \alpha_2)]^T$ for which the system gives maximum amplification. What is the output signal in this case?

4. Recall Fig. 1, which consists of the interconnected plant P and controller K, driven by the external signals w_1 and w_2.

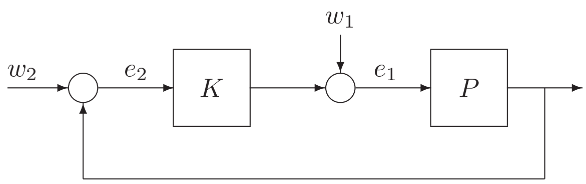

Figure 1: Standard feedback control configuration

Assume that P and K are fixed real rational proper TM with appropriate dimensions, and $K \in \mathcal{RH}_\infty$. Show that the above system is internally stable if and only if it is well-posed and $P(I - KP)^{-1} \in \mathcal{RH}_\infty$.