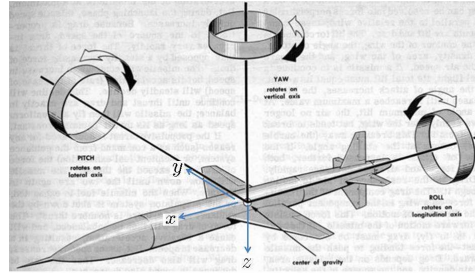


## Term Project Fall 2008

In missiledat.mat file, you will find nine sets of open-loop system matrices  $(A_{11}, B_{11}, C_{11}), \dots, (A_{33}, B_{33}, C_{33})$ . They represent dynamics of a skid-to-turn missile with  $x := [w \ q \ v \ p \ r]^T$  and  $u := [\delta_r, \delta_z, \delta_y]^T$  in various conditions, when the forward velocity along the  $x$  axis is fixed.



- $a_z, a_y$  = Normal and lateral acceleration  
 $p, q, r$  = Roll, pitch, and yaw rate  
 $v, w$  =  $y$  and  $z$  velocity components in the body coordinate frame  
 $\delta_r, \delta_z, \delta_y$  = Roll, pitch, and yaw fin deflection  
 $\phi, \theta, \psi$  = Roll, pitch, and yaw Euler angle

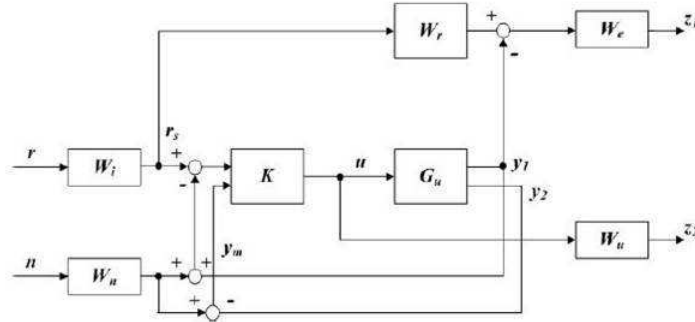
$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{v} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} Z_w & Z_q & Z_v & Z_p & 0 \\ M_w & M_q & M_v & M_p & 0 \\ Y_w & 0 & Y_v & Y_p & Y_r \\ L_w & 0 & L_v & L_p & 0 \\ N_w & 0 & N_v & N_p & N_r \end{bmatrix} \begin{bmatrix} w \\ q \\ v \\ p \\ r \end{bmatrix} + \begin{bmatrix} Z_{\delta_z} & 0 & 0 \\ M_{\delta_z} & 0 & 0 \\ 0 & 0 & Y_{\delta_y} \\ L_{\delta_z} & L_{\delta_r} & L_{\delta_y} \\ 0 & 0 & N_{\delta_y} \end{bmatrix} \begin{bmatrix} \delta_z \\ \delta_r \\ \delta_y \end{bmatrix} \quad (1)$$

There exist an actuator in each input channel, whose dynamics is represented by a 2nd order transfer function with the natural frequency of 30 Hz and damping coefficient of 0.7.

Our goal is to design a controller to track a step reference signal  $r$  using the output  $y = [y_1^T, y_2^T]^T$ .

$$r = \begin{bmatrix} a_{zd} \\ a_{yd} \\ \phi_d \end{bmatrix}, \quad y_1 = \begin{bmatrix} a_z \\ a_y \\ \phi \end{bmatrix}, \quad y_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (2)$$

It is desired for  $a_z, a_y$  and  $\phi$  to settle within 1 sec, with overshoot less than 50 % (if you can make them 0.5 sec and 25 %, even better).



1. If we ignore the cross-coupling effects, the equations of longitudinal motions can be represented by the first two columns and rows in Eq. (1) (i.e. involving  $w, q, \delta_z$ ). And the lateral motions are the last three columns and rows (i.e. involving  $v, p, r, \delta_r, \delta_y$ ).  
For now, assume that all the state variables are available for feedback. Decouple  $(A_{22}, B_2, C_{22})$  into the longitudinal and lateral channels. Design separate controllers for the longitudinal and lateral channels, and show that each channel is stabilized under your controller.  
Then, apply those two controllers together to the overall  $(A_{22}, B_2, C_{22})$ , and see whether the overall response is satisfactory or not.
2. Design an output feedback controller that takes  $y$  and shows satisfactory step response for as many cases as possible among  $(A_{11}, B_{11}, C_{11}), \dots, (A_{33}, B_{33}, C_{33})$ . Present your interconnection structure with weighting functions. Simulate the response for each system, for commands  $r = [10, 10, 0]^T$  and  $r = [10, -10, 0]^T$ .

It may be helpful to consider the interconnection structure given in Fig. Assume that the sensor noise weight is

$$W_n = nd2sys([2 \ .01 * 300], [1 \ 300]);$$