Aeroelasticity Assignment No. 4

Due: December 07 (Mon) 6:00 PM

Consider the typical section airfoil with two degrees of freedom as discussed in class. Assuming the 2-D incompressible strip aerodynamics, equations for the lift and pitch moment at the elastic axis are given by Eqs. 5-311 and 5-312 (B.A.H., p. 272), respectively.

1) Derive the equations of motion.

2) Assuming a solution of the form $\alpha = \bar{\alpha} e^{pt}$ and $h = \bar{h} e^{pt}$, where $p = \sigma + i\omega$, set up the flutter determinant.

3) Considering the following numerical parameters:

$$e = 0.4b,\ \frac{\omega_h}{\omega_\alpha} = 0.5,\ x_\alpha \equiv \frac{S_\alpha}{Mb} = 0.05,\ r_\alpha \equiv \sqrt{\frac{I_\alpha}{Mb^2}} = 0.5,\ \frac{2M}{\pi \rho b S} = 10$$

plot the roots (real and imaginary parts) of the characteristic equation as function of the reduced velocity ($\frac{U}{b\omega_\alpha}$) and identify the flutter point (including flutter mode) for the following cases:

3.1) Quasi-steady aerodynamics (with and without aero damping)

3.2) Quasi-unsteady aerodynamics

3.3) Unsteady aerodynamics

4) Repeat (3) for $\frac{\omega_h}{\omega_\alpha} = 0.1$ and $\frac{\omega_h}{\omega_\alpha} = 0.8$

5) Compare your solutions and discuss the importance of the different terms of the aerodynamic operator (i.e., aerodynamic mass, damping, and stiffness).

6) Use Equation 6-136 (B.A.) to determine the reduced flutter velocity for the above cases, and compare with the results obtained from (3) and (4). Comment on the eventual discrepancies.

힌트: 문제 3번의 3.1)과 3.2)는 오직 우변의 aerodynamics term을 경우에 따라 포함 또는 제외시키기 위한 구분임. (강의시간에 제시된 Undamped system는 좌, 우변의 damping 항들을 모두 무시한, 너무나 간단한 경우임을 주의바람.) 따라서 다음과 같이 다시 세분하여 문제 풀이를 진행하면 됨.

Hint: In Prob.3, the difference between 3.1) and 3.2) is the only consideration of the terms included in the aerodynamic formulation. Please be careful that the "undamped system” included in the lecture notes corresponds to quite simplified situation where all the damping terms included in both right- and left-hand sides are completely neglected. Thus please proceed to each case of the problem solving as follows.

3.1) Quasi-steady aerodynamics

Set C(k) = 1.

- no aerodynamic mass and no aerodynamic $\dot{\alpha}$ term
- with damping terms ($\dot{\alpha}$ and $\dot{h}$ terms) and apparent mass terms
- with damping terms ($\dot{\alpha}$ and $\dot{h}$ terms) but no apparent mass terms

3.2) Quasi-unsteady aerodynamics

Include C(k).

- only aerodynamic damping and stiffness terms, but no aerodynamic mass terms

3.3) Unsteady aerodynamics

Include all the terms and C(k).

그리고, 3.2)와 3.3)의 경우 Theodorsen 함수 C(k)의 정의에 의하여 원칙적으로 iterative한 계산절차를 밟아야 하나, 다음과 같은 근사적인 Pade approximation을 사용하면 식이 길어지기는 해도 iterative한 계산을 피할 수 있음.

In Case 3.2) and 3.3), you may take an iterative step by referring to the original definition of C(k). However, you may use the approximate formula, which is as follows, to avoid the iterative computation step.

$$C(\bar{p}, \bar{U}) = \frac{0.55\bar{p} + 0.15\bar{U}}{\bar{p} + 0.15\bar{U}},$$

$$\text{where} \sim \bar{p} = \frac{p}{\omega_\alpha}, \bar{U} = \frac{U}{b}$$