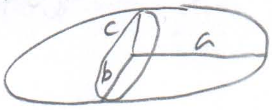
Homework 2.

1. 우선



$a \geq b \geq c \geq 0$ 인 ellipsoid 에 대해 각축 a, b, c 의 demag factor 를 L, M, N 이라 하자.

$$L = \frac{\cos\phi \cos 2\theta}{\sin^3\theta \sin^2\alpha}\left[F(k,\theta) - E(k,\theta)\right]$$

$$M = \frac{\cos\phi \cos 2\theta}{\sin^3\theta \sin^2\alpha \cos^2\alpha}\left[E(k,\theta) - \cos^2\alpha\, F(k,\theta) - \frac{\sin^2\alpha \sin\theta \cos\theta}{\cos\phi}\right]$$

$$N = \frac{\cos\phi \cos\theta}{\sin^2\theta \cos^2\alpha}\left[\frac{\sin\theta \cos\phi}{\cos\theta} - E(k,\theta)\right]$$

여기서 $\cos\theta = \frac{c}{a}$, $\cos\phi = b/a$ $(0 \leq \theta, \phi \leq \pi/2)$.

$$\sin\alpha = \left[\frac{1-(b/a)^2}{1-(c/a)^2}\right]^{1/2} = \frac{\sin\phi}{\sin\theta} = k \quad (0 \leq \alpha \leq \pi/2).$$

$F(k,\theta)$, $E(k,\theta)$ 는 elliptic integrals. k : modulas.

θ : amplitude of intergrals.

다양한 형태의 ellipsoid 의 L, M, N 구할 수 있다.

i) $b = c$

$$L = \frac{1}{m^2-1}\left[\frac{n}{2(m^2-1)^{1/2}} \times \ln\left(\frac{n+(m^2-1)^{1/2}}{m-(m^2-1)^{1/2}}\right) - 1\right]$$

$$M = N = \frac{m}{2(m^2-1)}\left[m - \frac{1}{2(m^2-1)^{1/2}} \times \ln\left(\frac{m+(m^2-1)^{1/2}}{M-(m^2-1)^{1/2}}\right)\right]$$

여기서, $m = \frac{a}{c}$

Homework 2 계속

$m \gg 1$ 이면

$$L \simeq (1/m^2)(\ln 2m - 1)$$

$$M = N \simeq \frac{1}{2}\left[1 - (\ln 2m - 1)/m^2\right]$$

ii) $a \gg b \geq c$

$$L = (bc/a^2)\left(\ln 4a/(b+c) - 1\right).$$

$$M = c/(b+c) - \frac{1}{2}(bc/a^2)\ln\left[4a/(b+c)\right] + bc(3b+c)/4a^2(b+c)$$

$$N = b/(b+c) - \frac{1}{2}(bc/a^2)\ln\left[4a/(b+c)\right] + bc(b+3c)/4a^2(b+c).$$

iii) $a = \infty$, $b \geq c$

$$M = c/(b+c)$$

$$N = b/(b+c).$$

iv) $a = b$

$$L = M = \frac{1}{2(m^2-1)}\left\{m^2(m^2-1)^{-1/2} \times \arcsin\left[(m^2-1)^{1/2}/m\right] - 1\right\}.$$

$$N = m^2/(m^2-1)\left\{1 - 1/(m^2-1)^{1/2} \times \arcsin\left[(m^2-1)^{1/2}/m\right] - 1\right\}$$

$m \gg 1$ 이면

$$L = M \simeq (\pi/4m)(1 - 4/\pi m).$$

$$N \simeq 1 - \pi/2m + 2/m^2$$

v) $a \geq b \gg c$.

$$L = \frac{c}{a}(1-e^2)^{1/2}\frac{K-E}{e^2}, \quad M = \frac{c}{a}\frac{E-(1-e^2)K}{e^2(1-e^2)^{1/2}}$$

$$N = 1 - \frac{cE}{a(1-e^2)^{1/2}}$$

K, E 는 elliptic integral $e = (1 - b^2/a^2)^{1/2}$.