Home problems #3 Topics of Magnetic Materials Due: Oct. 27 (Mon), 2008

1. Prove the relative permeability μ_r is given by

$\mu_r = 1 + \chi$ (suceptibility) in SI, and $\mu_r = 1 + 4\pi\chi$ in cgs.

2. A circular magnetic field is produced around the wire axis of a straight wire carrying the current i in a plane normal to the axis.

(a) Using both Biot-Savart law and Ampere's law, prove that outside the wire the magnitude of this field, at a distance r cm from the wire axis, is given by

$$H = \frac{i}{2\ r} \text{ [A/m]}$$

π

(b) Calculate the magnetic field H(in Oe) at r = 10 cm for i = 10 A.

3. (a) Calculate the magnetic induction B (in Tesla) and magnetization M (in A/m) of a paramagnetic material with the relative permeability $\mu_r = 1.001$ under an applied field strength of 5.0×10^5 A/m.

Where $\mu_0 = 4\times10^{-7}$ henry/m and 1 henry · A = 1 Wb.

(b) Calculate the magnetic susceptibility χ of this material.

4. For a circular loop of the radius a, the field at a point x on the axis of the circular coil is given by $H = ia^2/2(a^2 + x^2)^{3/2}$. If a circular coil of 100 turns and diameter 10 cm carries a current of 0.1 A, calculate the magnetic field (in A/m) at the center of the coil and at a distance of 50cm along the axis of the coil, respectively.

5. Calculate the magnetic induction $\boldsymbol{B}$ and flux Φ at the center of a toroidal solenoid with mean circumference of 50 cm and cross-sectional area of 2.0 cm^2, wound with 800 turns of wire carrying 1.0 A, (a) when the solenoid has an air core and (b) when the solenoid has a soft iron core of relative permeability 1000.

6. How strong is the applied magnetic field H_a (in Oe) needed to magnetize an iron sphere to its saturation magnetization ($M_s = 1.69\times10^6$ A/m)? Assume that the field needed to overcome the demagnetizing field H_d is much greater than the field needed to saturate the material in toroidal form. The demagnetizing factor N_d of a sphere is $4\pi/3$ in cgs (1/3 in mks).

7. A specimen has a length to diameter of 8:1. At a field strength of H_a = 80 kA/m the magnetic induction ***B*** is 0.9 tesla, while at a field strength of H_a = 160 kA/m the induction is 1.1 tesla. Assume the demagnetizing factor N_d is 0.02.

(a) Calculate the internal (or true) magnetic field H_{in} (= H_a - H_d) in each case.

(b) If we consider the fractional error in the observation of the uncorrected field, what do you conclude about this error at higher fields? (Is this error becomes larger or smaller at higher fields? Explain the reason.)

8. Answer the following questions.

(a) Prove that the demagnetizing factor N_d of a sphere is given by $4\pi/3$ in cgs (1/3 in mks).

(b) At a field strength of H_a = 80 kA/m, the magnetic induction B of a spherical sample is 1 Tesla. Calculate the internal (or true) magnetic field H_{in} (= H_a - H_d) of this sample.

(c) How strong is the applied magnetic field H_a needed to magnetize an iron sphere to its saturation magnetization (M_s = 1.69×10^6 A/m)? Assume that the field needed to overcome the demagnetizing field H_d is much greater than the field needed to saturate the material in toroidal form.

9. Explain the origin of demagnetization field and its effect on the $M(H)$ curve of a ferromagnet material.

10. Discuss the values of μ_r and χ for the following cases.

i) normal diamagnetic materials
ii) superconducting materials
iii) paramagnetic and antiferromagnetic materials
iv) ferromagnetic and ferrimagnetic materials

11. Draw and explain χ (T) for normal diamagnets, both normal and Pauli paramagnets, and antiferromagnets. Also, draw $M(T)$ curves for normal ferromagnets and ferrimagnets below T_c.

12. (a) Draw $1/\chi$ or σ_s versus T (in K) for five main kinds of magnetism. On

the graphs, express T_N and T_c.,
(b) Draw the arrangement of the net atomic magnetic moment as an arrow on a circle representing an atom or ion for each case in (a).

13. Answer the following questions.
(a) Derive the relationship between the Weiss molecular field H_e of a ferromagnet and the Curie temperature T_c. (b) Calculate the value of H_e for Ni which has a Curie temperature of 358℃ and a net magnetic moment per atom of 0.604 μ_B.

14. Answer the following questions.
(a) Using classical Boltzman statistics, prove that M/M_o = coth$a$ $- a$ = $L(a)$,
where $a = \dfrac{\mu_o m H}{kT}$ and $L(a) = \dfrac{a}{3} - \dfrac{a^2}{45} + \dfrac{2a^5}{945} + \cdots$
(b) Draw $L(a)$ versus a and explain two important conclusions from the Langevin theory for paramagnetism.
(c) Derive the Curie law from the Langevin theory: $\chi = \dfrac{C}{T}$, $C = \dfrac{N\mu_o m^2}{3k}$
(d) Using the Weiss assumption of the molecular field (or exchange field), $\boldsymbol{H}_e = a\boldsymbol{M}$, where a is the molecular field constant, derive the Curie-Weiss law from the Curie law in (a) : $\chi = \dfrac{C}{T-\theta}$, $\theta = aC$

15. Quantum Theory에 의하면 강자성체가 paramagnetism을 나타내는 고온에서 $x\{= gJ\mu_B\mu_0(H+\alpha M)/k_BT\}$가 매우 작으므로 Brillouin function, $B_J(x) = (J+1)x/3J$의 근사식으로 나타낼 수 있다. 이 관계식으로부터 Curie-Weiss law를 유도하여 Curie 상수 C와 Curie 온도 T_c는 다음과 같이 주어짐을 증명하라.
$C = Ng^2\mu_B^2J(J+1)/3k_B$, $T_c = \alpha N\mu_0 g^2\mu_B^2J(J+1)/3k_B$.

16. Using quantum theory, prove the relationships.
(a) $M/M_o = \dfrac{2J+1}{2J} coth\left(\dfrac{2J+1}{2J}\right)x - \dfrac{1}{2J}\coth\dfrac{x}{2J}$, where $x = \dfrac{gJ\mu_B\mu_o H}{kT}$
(b) If $J = \dfrac{1}{2}$ (one spin /atom), $\dfrac{M}{M_o}$ = tanh$x$

(c) In the case of Ni, $J = S = \frac{1}{2}$, thus $\frac{M_s}{M_o} = \tanh(\frac{\mu_B \mu_o \alpha M_s}{kT})$ in the absence of a magnetic field since $\boldsymbol{B} = \mu_o(\boldsymbol{H} + \alpha \boldsymbol{M}_s)$ in ferromagnetic materials.

On the basis of this equation, explain the temperature dependence of $\boldsymbol{M}_s$

17. Answer the following questions.

(a) Show that Brillouin function $B_J(x)$ reduces to the Langevin function $L(x)$, with $x = mB/k_BT$, in the limit J approaches infinity.

(b) Derive the Curie law from Lagevin function at high temperatures

(c) Show that $B_J(x) = \tanh(x)$ for $J = 1/2$

(d) Show that $B_J(x) = \{J(J+1)/3J^2\}x$ and

thus susceptibility $\chi = \mu_0\{Ng^2\mu_B^2 J(J+1)/3k_BT\}$ for $x \ll 1$,

(e) Derive the Curie-Weiss law in the paramagnetic region from the above relations and express Curie constant and Curie temperature for

$x = gJ\mu_B\mu_0(H+\alpha M)/k_BT$