Elementary Numerical Analysis

2008 년 2 학기

HW#2: Polynomial Interpolation

Due Sept. 18

In this problem set, you will examine the two methods of polynomial interpolation and the error associated with using a finite order polynomial.

1. Show that the (n+1)th order divided difference is related with the (n+1)th order derivative as follows:

$$f[x, x_0, x_1, ...x_n] = \frac{f^{(n+1)}(\xi)}{(n+1)!}; \quad a \le \xi(x) \le b$$

   where all x points are defined within [a,b]. Use the fact that the error of an n-th order polynomial approximating *f(x)* is:

$$E(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^{n} (x - x_i)$$

2. Write a function program that finds the coefficients of the Lagrange polynomial given two vectors specifying the coordinates of the x and y points. Write another function that obtains the polynomial value at a specified point. Use the x-value, data point x-coordinate vector, the coefficient vector as the input argument.

3. Perform Lagrange interpolation with the following 11 data points which represent a damped oscillation given by

$$f(x) = e^{-2x} \cos(4\pi\ x)$$

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x: 0.0        0.1         0.2        0.3         0.4        0.5
y: 1.0000e+0  2.5300e-1 -5.4230e-1 -4.4400e-1  1.3885e-1  3.6788e-1
x: 0.6        0.7         0.8        0.9         1.0
y: 9.3074e-2 -1.9950e-1 -1.6334e-1  5.1080e-2  1.3534e-1
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   Evaluate the polynomial at 101 points (interval size 0.01) and plot the results along with the true function.

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4. Suppose that there is one more data point available, (0.85, -5.6452e-2). You can use this additional point to obtain the (n+1) the order divided difference which can be used to estimate the error using the formula given in the second equation in Prob. 1. Estimate the error at x=0.95 by estimating the divided difference using the attached function. Compare this estimated error with the exact error obtained from the known function value.