Elementary Numerical Analysis

2008 년 1 학기

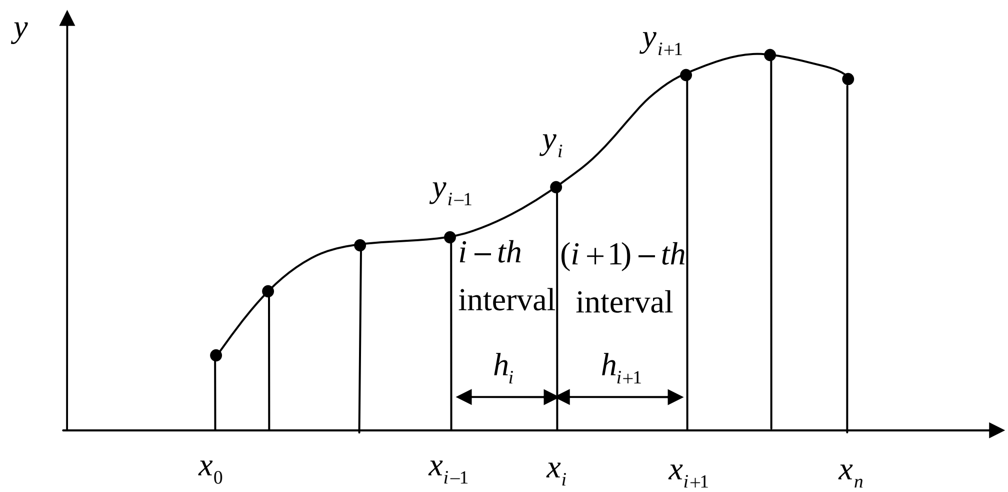
HW#3: Cubic Spline

Sept. 23, 2008

Cubic spline is to fit given data points using piecewise third-order (cubic) polynomials. If there are N+1 data points, there are N intervals and thus N cubic polynomials need to be defined, which require 4N coefficients. Let the polynomial for the *i*-th interval be

$$p_i(x) = a_i + b_i(x - x_{i-1}) + c_i(x - x_{i-1})^2 + d_i(x - x_{i-1})^3 \ , \ x_{i-1} \le x \le x_i ; \ h_i = x_i - x_{i-1} .$$

for the following setup:



The constant term above can be determined easily by requiring the function value at the left end point of the interval be the given value at the point, namely, $a_i = y_{i-1}$. So there are three unknown coefficients to be determined per interval or 3N unknowns for the N intervals. Now, the requirement for the function value at the right end point leads to

$$p_i(x_i) = y_{i-1} + b_i h_i + c_i h_i^2 + d_i h_i^3 = y_i \ .$$

The above equation holds for the N intervals (i=1 to N) so that there N equations are available. In order to determine the 3N unknowns, we need 2N more equations, a part of which can be obtained from the continuity conditions for the first and second order derivatives at the interior points. Since there are N-1 interior points, two continuity conditions per interior point lead to 2(N-1) equations and we thus lack 2 equations. The two equations can be obtained by specifying the first order derivative (or slope) at the two end points (x_0, x_N). Now answer the following:

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1. Give the continuity conditions for the first and second order derivatives.
2. Give the conditions for the slopes at the two end points. Pretend that the slopes at the two end points are known.
3. Construct a linear system consisting of the 3N equations for the 3N unknowns. Arrange the unknowns in the following order: $b_1,c_1,d_1,b_2,c_2,d_2, \ldots, b_N,c_N,d_N$ and the equations in the following order:

   Slope Condition at x_0

   $$\left.\begin{array}{l}\text{Function value at } x_i \\ \text{First Order Derivative Continuity at } x_i \\ \text{Second Order Derivative Continuity at } x_i\end{array}\right\} for\ i = 1 : N-1$$

   Function Value at x_N

   Slope Condition at x_N

   Sketch the structure of the linear system (indicate the non-zero entries of the matrix and right hand side vector with the symbol x).
4. Write a MATLAB function to construct and solve the above linear system. Use the MATLAB x=A\b left division function to solve the linear system. Suppose that the x and y points are specified by the two vectors, XN and YN, and the slopes at the two end points are given as YPL and YPR. Use these two vectors and two values as the input parameters and return the 3N coefficients in terms of three vectors BN, CN, DN as the output.
5. Write a MATLAB function to determine the slope at the left end points using the first three data points and then to determine the slope at the right end points using the last three data points. You need to derive the derivative using the second order Lagrange polynomial.
6. Use the two MATLAB functions to perform the cubic spline fitting for the data set given in the HW#2 consisting of 11 data points. Plot the fitted function along with the data points.
7. Given the true function, compare the errors of the cubic spline fit and the 10-th order polynomial fit you obtained in HW#2 at points 0.01 through 0.99 with the increment of 0.01. Try also the true derivatives at both ends. Discuss your results.