Elementary Numerical Analysis

2008 년 2 학기

HW4: Approximation of Functions

Oct 7, 2006

A. Best m-th order polynomial fit

For a function defined on interval [a, b], you are going to find an m-th order polynomial which minimizes the maximum error within this interval. You can achieve your goal by suitably choosing the (m+1) interpolation points within the interval using the Chebyshev polynomial property as discussed in the class. Step through the following procedure first. Then answer the rest of the questions. Perform all the work by writing proper programs.

1. Write a (MATLAB) function that determines those m+1 points within the interval.

2. Obtain the m+1 points and the function values at those m+1 points for
    a. $f(x) = xe^x$ in [0, 1.5] with m=4
    b. $f(x) = \cos(\ln x)$ in [1, 3] with m=5

3. Determine the (m+1) coefficient of each term appearing in the Lagrange interpolation (Use your own program or the MATLAB script provided – lagp0.m)

4. Divide the interval into 1000 subintervals and evaluate the polynomial and the function at 1001 points and then determine the maximum absolute error you're your own or lagp.m).

5. Choose the m+1 points in the following ways to determine the m-th order polynomial. Compare then the maximum error of each case with the one obtained from 4 to show that the Chebyshev polynomial based points give indeed the lowest maximum error.
    a. Equidistance
    b. 10 Different sets of random points (use the rand function-rand(m+1,1))

B. Pade Approximation of Matrix Exponential

Pade approximation can be useful in calculating matrix exponentials. Use the Pade approximation program for the exponential function, exppade.m, discussed during the class to analyze the error of matrix exponential of two matrices:

$$A = \begin{bmatrix} 1 & 0.5 \\ -0.5 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ -1 & 5 \end{bmatrix}$$

1. Explain the algorithm implemented in exppade.m
2. Use different pairs of (n,m) and observe the error of Pade approximation of the matrix exponential until you obtain the error in each component of the matrix exponential less than 0.5%. In MATLAB, matrix exponential can be readily obtained by expm(A).
3. Matrix B would require higher orders. What would be the major factor requiring higher orders in Pade approximation of matrix exponential?