Elementary Numerical Analysis

2008 년 2 학기

HW#5: 1-D Particle Diffusion and LU Factorization

Oct. 21, 2008

Part 1. 1-D Particle Diffusion

In this problem set, you are going to find the behavior of the numerical solution of the 1-D particle diffusion given by:

$$-D\frac{d^2\phi(x)}{dx^2}+\sigma\phi(x)=s(x),\quad x\in[0,a] \qquad (1)$$

with the reflective boundary condition on the left boundary and the zero flux boundary condition on the on the right boundary where the physical constants are as follow:

$$a=10\ cm$$
$$D=2.0\ cm$$
$$\sigma=0.5\ cm^{-1}$$
$$s(x)=\begin{cases} 1 \\ or \\ 1-0.2|x-5| \end{cases} \quad particles/cm^3/\sec$$

The 10 cm domain is divided into 10 subdomains with an equal width, namely, 1cm. We are going to obtain the total flux at each subdomain (10 total fluxes) as well as the total flux for the whole domain.

A. Uniform Source

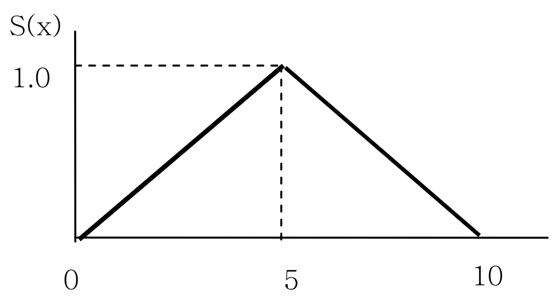
With the uniform source of unit strength, it is possible to find the analytic solution of the problem in terms of $\cosh(Bx)$ as discussed in the class. Use the analytic solution to examine the behavior of the error for various mesh sizes as follows:

1. Let $n=10\cdot 2^k$ be the total number of meshes at the k-th step with k=0 being the base step. Write a MATLAB (or any other language) program to construct and solve the linear system for the discretized equations with k as a parameter. Use the normal form of Eq. (1) for discretization.
2. Define a vector consisting of the ten subdomain fluxes and determine the vector elements from the solution at the k-th step. To obtain the subdomain flux, you need

to sum up the fluxes belonging to each subdomain and multiply the sum by the mesh size. Also obtain the total flux for the entire domain. For the analytic solution, define the subdomain vector in the same way, namely the summation of the fluxes obtained at the same discretized points as the numerical solution. Do not bother with integration. Define the error vector as the difference between the numerical subdomain flux vector and the analytic subdomain flux vector. And define the total flux error as the difference between the numerical total flux and the analytic total flux. Then observe the error reduction behavior at each step. See if you see indeed the error reduction by 1/4 at each step. Proceed until the error reduction ratio falls within 0.25 ± 0.0001? Plot the flux distribution at the base step and the final step at the same graph.

B. Non Uniform Source

For the nonuniform source distribution which appears as follows:



the analytic solution can not be obtained, hence the true solution needs to be predicted. Now follow the same steps as the above except using previous step solution instead of the analytic solution. What would be the predicted value of the true total flux? Use the extrapolation scheme discussed in the classes.

Part 2. LU-Factorization

1. Write a MATLAB function to perform LU factorization of a given square matrix of Rank *n* with the following features. (*n* is arbitrary)

1) Partial Pivoting
2) Produce the permutation matrix (P) as well as L and U such that PA=LU

Elementary Numerical Analysis

2008 년 2 학기

Verify your program by comparing with the MATLAB's function *lu* which gives [L,U,P]=lu(A) using a random matrix which can be formed by A=rand(n).

2. Write a MATLAB program to solve a linear system with LU factors by forward and backward substitution. Verify your program by comparing with the solution obtained with MATLAB's x=A\b function.