Elementary Numerical Analysis

2008 년 2 학기

HW#8: Roots of Legendre Polynomial

Due Dec 4, 2008

In this problem set, you will examine the various methods for fining roots of nonlinear functions. The test problem is the 8-th order Legendre polynomial which reads:

$$P_8(x) = \frac{35}{128} - \frac{315}{32}x^2 + \frac{3465}{64}x^4 - \frac{3003}{32}x^6 + \frac{6435}{128}x^8.$$

The objective is to find all the four positive roots of this polynomial.

1. Define the polynomial for the x range of [-1,1] and make estimates of the roots by eyes. Also divide the positive domain [0,1] into four intervals such that each interval include one root.

2. Write a MATLAB program to implement
   1) Bisection Method
   2) Modified Linear Interpolation Method (Secant Method)
   3) Newton-Raphson Method

3. Apply these three methods to find the four roots and compare the number of trials of these methods for each root. Use the absolute convergence criterion of 1.0×10^{-10} for both x and y. For the Newton-Raphson method choose the middle point of the interval as the starting point.

4. In this case, it is easy to guess the range where the roots are located. But in general the location of the roots is not known. In the general case, how would you determine the initial points? Discuss your logic for finding the first root. Then discuss how you would use the roots you already found to avoid finding those roots in the subsequent root finding. Apply your new method to determine all four roots with the Newton-Raphson method without the prior determination of the range of the roots already done in Prob 1.