11.31 (33)

$v = v'\sin(\omega_n t + \phi)$

Show (a) $v' = \dfrac{v_0^2 + x_0^2\omega_n^2}{2x_0\omega_n}$

$t=0$, $v_0$ and $x_0$

at Max Velocity (b) $x = x_0 \dfrac{3-(v_0/x_0\omega_n)^2}{2}$

Max displacement : $2x_0$

---

(a) Let $x$ be maximum at $t=t_1$ when $v=0$

$$\rightarrow v = v'\sin(\omega_n t+\phi)$$

$$0 = \sin(\omega_n t_1 + \phi)$$

$$v = \frac{dx}{dt} \rightarrow x = \int v\,dt$$

So, $x = -\dfrac{v'}{\omega_n}\cos(\omega_n t+\phi) + C$ — ①

At $t=0$, $x = x_0$

$$x_0 = -\frac{v'}{\omega_n}\cos\phi + C \quad \therefore C = x_0 + \frac{v'}{\omega_n}\cos\phi \quad \text{— ②}$$

① & ② $\quad x = x_0 + \dfrac{v'}{\omega_n}\cos\phi - \dfrac{v'}{\omega_n}\cos(\omega_n t+\phi)$ — ③

Max displacement is $2x_0$.

So, $x_{max} = 2x_0 = x_0 + \dfrac{v'}{\omega_n}\cos\phi + \dfrac{v'}{\omega_n}$

($\because$ when $\cos(\omega_n t+\phi) = -1$, $x$ becomes Max)

Solving for $\cos\phi$ $\quad \cos\phi = \dfrac{x_0\omega_n}{v'} - 1$

Using $\sin^2\phi + \cos^2\phi = 1$ (at $t=0$ $v_0 = v'\sin\phi$)

$$\left(\frac{v_0}{v'}\right)^2 + \left(\frac{x_0 w_n}{v'} - 1\right)^2 = 1$$

therefore $$v' = \frac{v_0^2 + x_0^2 w_n^2}{2x_0 w_n}$$

(b) When the velocity is maximized,

the acceleration equals zero.

$$a = \frac{dv}{dt} = v' w_n \cos(w_n t + \phi)$$

Let $v$ be maximum at $t = t_2$

then, $\cos(w_n t_2 + \phi) = 0$

From eqn. ③

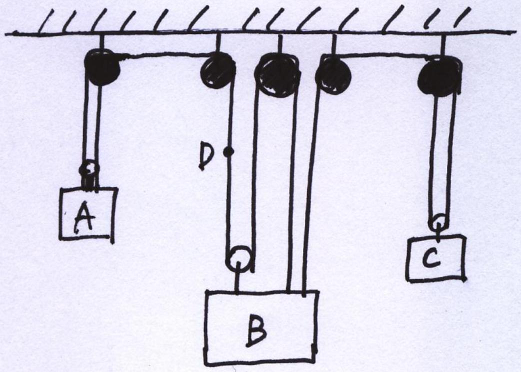
$$x = x_0 + \frac{v'}{w_n}\cos\phi \qquad \left(\text{when } \cos\phi = \frac{x_0 w_n}{v'} - 1\right)$$

$$= 2x_0 - \frac{v'}{w_n}$$

$$= x_0 \frac{3 - (v_0/x_0 w_n)}{2}$$

11.50 7th ed.

Find constant walue.




i) $2x_A + 3x_B = \text{constant}$ $\quad v_A = -\frac{3}{2} v_B$

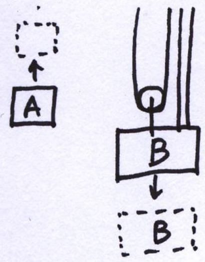
ii) $x_B + 2x_C = \text{constant}$ $\quad v_C = -\frac{1}{2} v_B$

iii) $2x_A + x_D = \text{constant}.$ $\quad v_D = -2v_A$

$v_B = 610 \text{ mm/s}$

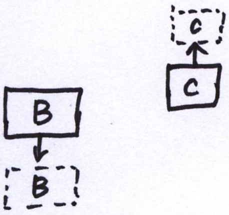
---

(a)



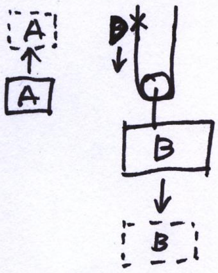

$v_A = 915 \text{ mm/s} \uparrow$

(b)



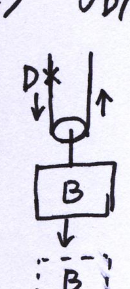

$v_C = 305 \text{ mm/s} \uparrow$

(c)




$v_D = 1830 \text{ mm/s} \downarrow$

(d) $v_{D/B} = ?$

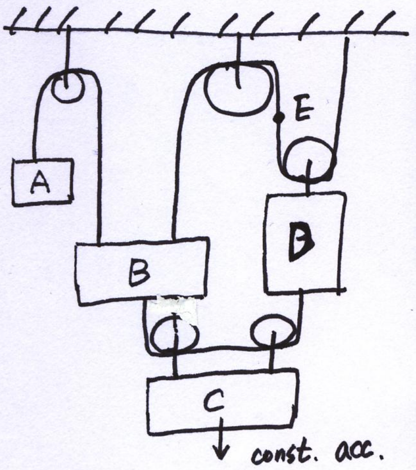



$$v_{D/B} = v_D - v_B$$

$$= 1830 - 610$$

$$v_{D/B} = 1220 \text{ mm/s} \downarrow$$

11.52 8th ed.




Find constant value of this system.

i) $x_A + x_B = \text{const.}$ $\quad v_B = -v_A \quad a_B = -a_A$

ii) $x_B + 2x_D = \text{const.}$ $\quad v_D = -\frac{1}{2}v_B \quad a_D = -\frac{1}{2}a_B$

iii) $(x_C - x_B) + (x_C - x_D) = \text{const.}$

$2v_C + \frac{1}{2}v_A = 0 \qquad v_C = -\frac{1}{4}v_A$

$a_C = -\frac{1}{4}a_A$

I.C. $\overset{t=5}{v_{A/D}} = v_A - v_D = \frac{1}{2}v_A = 2.4\,\text{m/s}$

(a) $a_C = ?$ $\quad a_C = -\frac{1}{4}a_A$

$a_A = 2a_{A/D}$

$v_{A/D} = a_{A/D} \times t$ $\quad$ so, $a_{A/D}$ at $t=5$

$a_{A/D} = 2.4/5$

$= 0.48\ \text{m/s}^2$

so, $a_A = 0.96\ \text{m/s}^2$

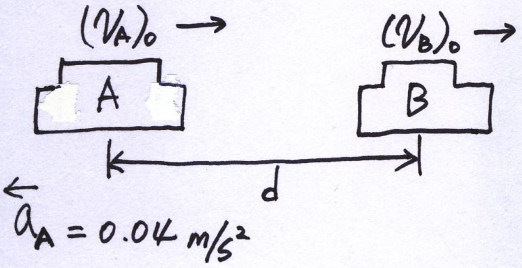
$\therefore\ a_C = 0.24\ \text{m/s}^2 \downarrow$

(b) $x_B + x_E = \text{const.}$ $\quad a_E = -a_B$

$= a_A$

so, $a_E = 0.96\ \text{m/s}^2 \uparrow$

11.73(75)

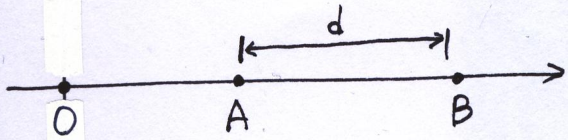



$(v_A)_0 = 36\ km/h$

$= 10.0\ m/s$

$(v_B)_0 = 20\ km/h$

$= 5.56\ m/s$




$(x_B)_0 - (x_A)_0 = d$

Calculate the time when $v_A = v_B$

$$v_A = (v_A)_0 - at = v_B$$

$$10 - 0.04t = 5.56$$

$$t = 111\ sec$$

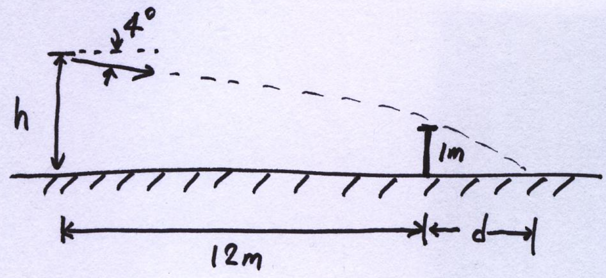
$$x_B = (x_B)_0 + (v_B)_0 t$$

$$x_A = (x_A)_0 + (v_A)_0 t - 0.02t^2$$

$$d = (x_B)_0 - (x_A)_0 = [(v_A)_0 - (v_B)_0]t - 0.02t^2 \quad (\because x_B = x_A)$$

$$= 246.42\ m$$

11. 104(106)




$V_0 = 36$ m/s

ball clears the net by 0.15 m

$(V_x)_0 = 36\cos 4° = 35.91$ m/s

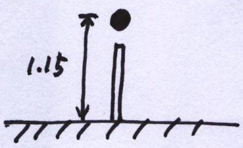
$(V_y)_0 = 36\sin 4° = 2.51$ m/s ↓

(a) $h = ?$ Find the time when the ball reaches to the net.

The ball travels 12 m through the x-axis.

$x = (V_x)_0 t \qquad \therefore\ t = \dfrac{x}{(V_x)_0} = 0.33$ s

$y = 1.15$ m




$h = y + (V_y)_0 t + \frac{1}{2}gt^2$ (be careful sign convention)

$= 2.51$ m

(b) when the ball lands, y equals zero. $y = 0$

$0 = h - (V_y)_0 t - \frac{1}{2}gt^2$

$4.9t^2 + 2.51t - 2.51 = 0$

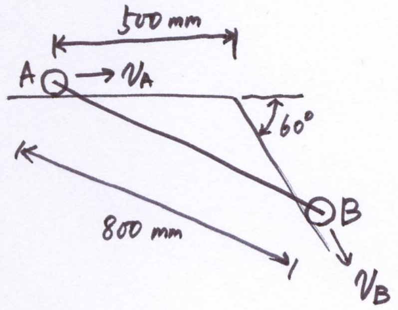
$t = 0.504$

$\therefore\ x = 35.91 \times 0.504$

$= 18.1$ m

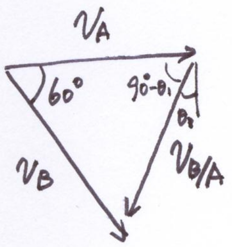
$d = 6.1$ m

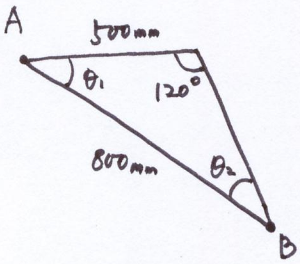
11. 118(122)




$v_A = 1.5\ m/s$

(a) $v_{B/A} = ?$




Using law of sines

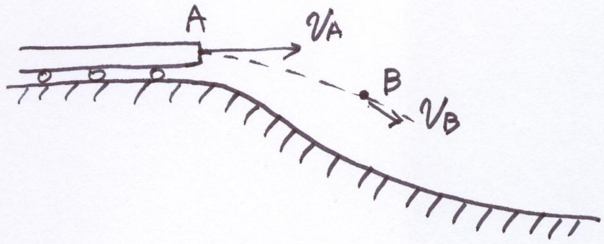
$$\frac{\sin\theta_2}{500} = \frac{\sin 120^\circ}{800} \qquad \therefore \theta_2 = 32.77^\circ$$

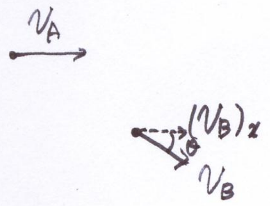
$$\therefore \theta_1 = 27.23^\circ$$

$$v_{B/A} = \frac{v_A \sin 60^\circ}{\sin 57.23^\circ} = 1.54\ m/s \quad 62.8^\circ$$

(b) $$v_B = \frac{v_A \sin 62.77^\circ}{\sin 57.23^\circ} = 1.59\ m/s \quad 60^\circ$$

11. 148 (150)

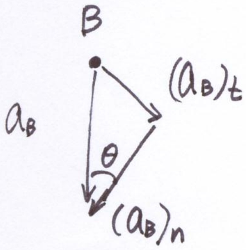



$$(v_B)_x = v_A$$

$$v_B \cos\theta = v_A$$

$\rho$ will define $\dfrac{v^2}{a_n}$



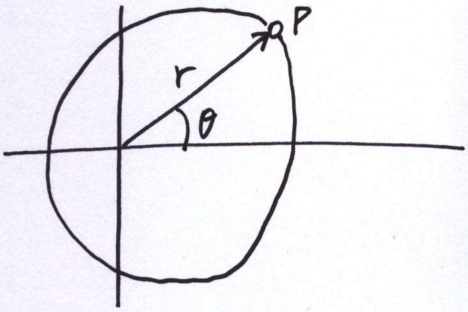

$$(a_B)_n = a_B \cos\theta$$

where $a_B = g$ , $\cos\theta = \dfrac{v_A}{v_B}$

Therefore $(a_B)_n = g \dfrac{v_A}{v_B}$

$$\therefore \rho = \frac{v_B^2}{(a_B)_n} = \frac{v_B^3}{g\, v_A}$$

11.164(166)




$r = b(2+\cos\pi t)$

$\theta = \pi t$

$\dot{r} = -\pi b \sin\pi t$

$\ddot{r} = -\pi^2 b \cos\pi t$

$\dot{\theta} = \pi$

$\ddot{\theta} = 0$

---

(a) At $t = 2s$ $v$ & $a$ ? $\sin\pi t = 0$ $\cos\pi t = 1$

$$v = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$a = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

$$v = 3\pi b\,\vec{e}_\theta$$

$$a = -4\pi^2 b\,\vec{e}_r$$

(b) Values $\theta$ for $|v_{max}|$

the magnitude of $v$ will be

$$|v| = \sqrt{v_r^2 + v_\theta^2}$$

$$v^2 = \pi^2 b^2 \sin^2\pi t + \pi^2 b^2 (2+\cos\pi t)^2$$

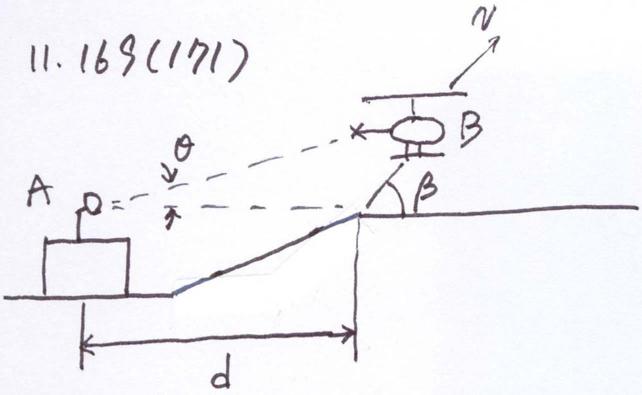
$$= \pi^2 b^2 [\sin^2\pi t + \cos^2\pi t + 4\cos\pi t + 4]$$

$$= \pi^2 b^2 [4\cos\pi t + 5]$$

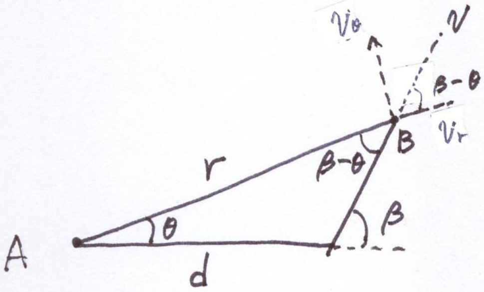
when $\cos\pi t = 1$, $v^2$ is maximized.

Therefore $\theta = 2n\pi$, $n = 0, 1, 2 \cdots$

11.169(171)




Using law of sines




$$\frac{r}{\sin(180^\circ-\beta)} = \frac{d}{\sin(\beta-\theta)}$$

$$\therefore\ r = \frac{d\sin\beta}{\sin(\beta-\theta)}$$

$$v = \dot{r}\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta$$

$$v_\theta = r\dot{\theta} \quad \& \quad v_\theta = v\sin(\beta-\theta)$$

$$r\dot{\theta} = v\sin(\beta-\theta)$$

$$\therefore\ v = \frac{r\dot{\theta}}{\sin(\beta-\theta)}$$

$$= \frac{d\sin\beta}{\sin^2(\beta-\theta)}\,\dot{\theta}$$