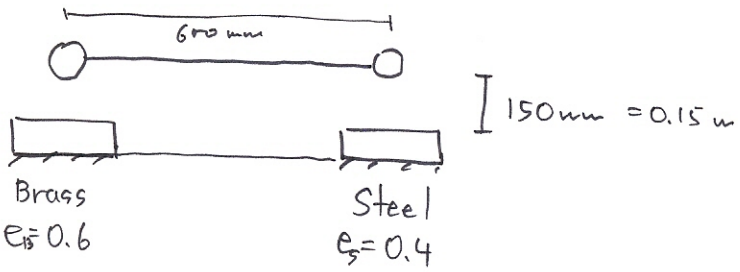
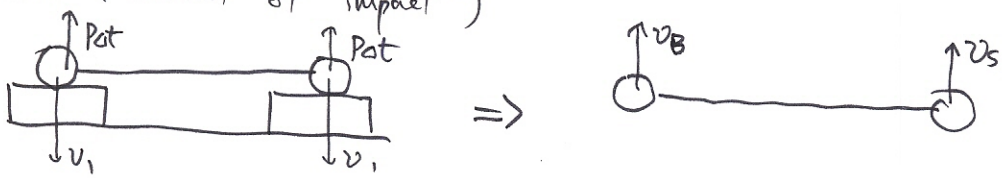
Homework #4 solution

1.



FBD (moment of Impact)



$$v_1 = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 0.15} = 1.715 \text{ m/s}$$

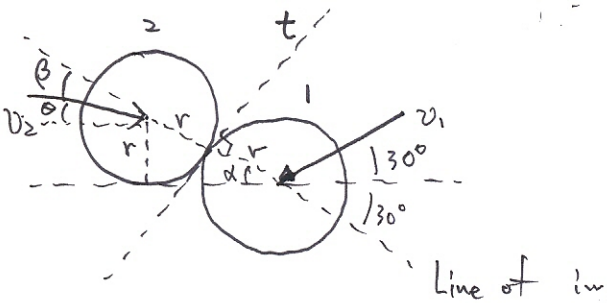
$$v_B = e_B v_1 = 0.6 \cdot 1.715 = 1.029 \text{ m/s}$$

$$v_S = e_S v_1 = 0.4 \cdot 1.715 = 0.686 \text{ m/s}$$

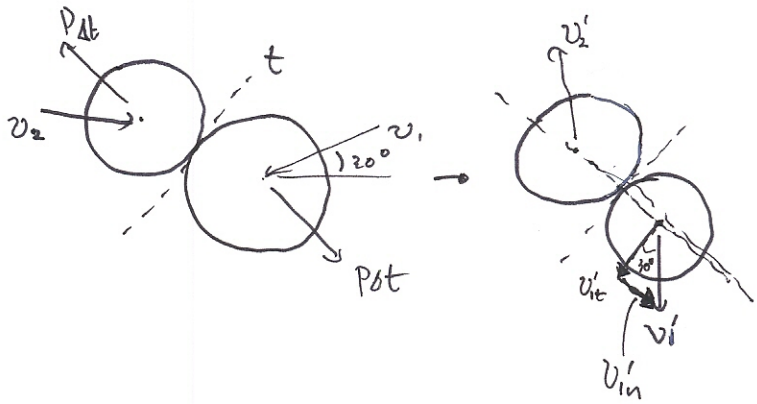
$$\dot{\theta} = \frac{v_S - v_B}{r} = \frac{1.029 - 0.686}{0.6} = \underline{0.572 \text{ rad/s}} \quad //$$

2.

FBD



$\alpha = 30°$



- Conservation of normal momentum

$$m(-v_1\cos 60) + m(v_2\cos\beta) = mv_{1n}' + mv_{2n}' \quad (v_1 = v_2) \cdots ①$$

(a) $e = 0.8$

$$e = 0.8 = \frac{v_{2n}' - v_{1n}'}{-v_1\cos 60 - v_2\cos\beta} \cdots ②$$

$$①' \rightarrow v_1(-\cos 60 + \cos\beta) = v_{1n}' + v_{2n}'$$

$$①', ② \rightarrow v_{1n}' = v_1(0.9\cos\beta - 0.05)$$

$$v_{1t}' = v_{1t} = v_1\sin 60 = \frac{\sqrt{3}}{2}v_1$$

$$\frac{v_{1n}'}{v_{1t}'} = \tan 30° = \frac{v_1(0.9\cos\beta - 0.05)}{\frac{\sqrt{3}}{2}v_1} = \frac{1}{\sqrt{3}}$$

$$\cos\beta = 0.611,$$

$$\therefore \beta = \pm 52.33°$$

$$\therefore \begin{cases} \theta_1 = 30 + 52.33 = \underline{82.33°}, \\ \theta_2 = 30 - 52.33 = \underline{-22.33°}, \end{cases}$$

(b) $e = 0.9$

$$e = 0.9 = \frac{v_{2n}' - v_{1n}'}{-v_1\cos 60 - v_2\cos\beta} \quad \cdots ②'$$

①, ②' → $v_{1n}' = v_1(0.95\cos\beta - 0.025)$

$$v_{1t}' = \frac{\sqrt{3}}{2}v_1$$

$$\frac{v_{1n}'}{v_{1t}} = \tan 30 = \frac{v_1(0.95\cos\beta - 0.025)}{\frac{\sqrt{3}}{2}v_1} = \frac{1}{\sqrt{3}}$$

$$\cos\beta = 0.526$$

$$\therefore \beta = \pm 58.24^\circ$$

$$\begin{cases} \theta_1 = 30^\circ + 58.24^\circ = 88.24^\circ \\ \theta_2 = 30^\circ - 58.24^\circ = -28.24^\circ \end{cases} //$$

(c) $e = 0.7$

$$e = 0.7 = \frac{v_{2n}' - v_{1n}'}{-v_1\cos 60 - v_2\cos\beta} \quad \cdots ②'$$

①, ②' $v_{1n}' = v_1(0.85\cos\beta - 0.075)$

$$v_{1t}' = v_{1t} = v_1\sin 60 = \frac{\sqrt{3}}{2}v_1$$

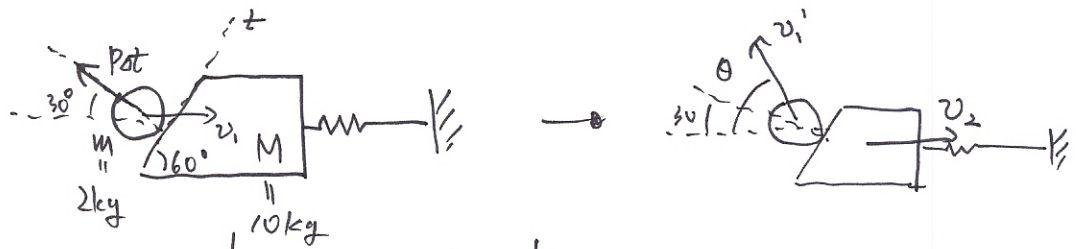
$$\frac{v_{1n}'}{v_{1t}'} = \tan 30 = \frac{v_1(0.85\cos\beta - 0.075)}{\frac{\sqrt{3}}{2}v_1} = \frac{1}{\sqrt{3}}$$

$$\cos\beta = 0.676$$

$$\beta = \pm 47.43^\circ$$

$$\begin{cases} \theta_1 = 30 + 47.43 = 77.43^\circ \\ \theta_2 = 30 - 47.43 = -17.43^\circ \end{cases} //$$

3. FBD



• momentum x-direction

$$mv_1 + 0 = -mv_1'\cos\theta + Mv_2$$

$$2(10) = -2v_1'\cos\theta + 10v_2 \quad \cdots ①$$

• t-direction for sphere.

$$mv_1\sin 30 = mv_1'\sin(\theta-30)$$

$$20\sin 30 = 2v_1'\sin(\theta-30) \cdots ②$$

• $e = 0.6$

$$0.6 = \frac{v_2\sin 60 + v_1\cos(\theta-30)}{v_1\cos 30} \quad \cdots ③$$

$$\rightarrow 5.196 = 0.866v_2 + 0.866v_1'\cos\theta + 0.5v_1\sin\theta$$

①③ $5.196 = 0.866(2 + 0.2v_1'\cos\theta) + 0.866v_1'\cos\theta + 0.5v_1'\sin\theta$

$$\rightarrow 1.039v_1'\cos\theta + 0.5v_1'\sin\theta = 3.464 \cdots ④$$

② $\rightarrow 0.866v_1'\sin\theta - 0.5v_1'\cos\theta = 5 \quad \cdots ⑤$

④,⑤ $\rightarrow v_1' = 6.04\ m/s,\ \theta = 85.9°$

∴ ① $\rightarrow v_2 = 2.087\ m/s$

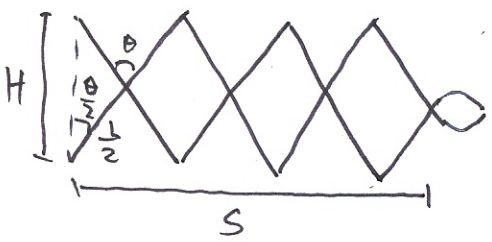
Energy conserve in spring

$$\frac{1}{2}Mv_2^2 = \frac{1}{2}k(\Delta x)^2$$

$$\frac{1}{2}\cdot 10\cdot 2.087^2 = \frac{1}{2}\cdot 1600(\Delta x)^2$$

$$\Delta x = 0.165\ m = \underline{165\ mm}$$

4.



$H = L\cos\frac{\theta}{2} \qquad S = \frac{\pi}{2} L \sin\frac{\theta}{2}$

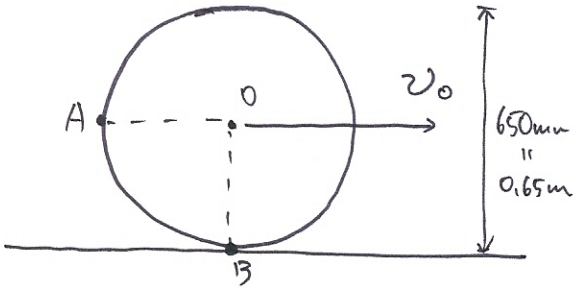
$\dot{H} = -u = -\frac{L\dot{\theta}\sin\frac{\theta}{2}}{2}$

$\dot{\theta} = \frac{2u}{L\sin\frac{\theta}{2}}$

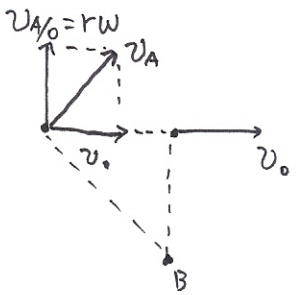
$\dot{S} = v = \frac{\pi}{4} L\dot{\theta}\cos\frac{\theta}{2}$

$= \frac{\pi}{4} L\left(\frac{2u}{L\sin\frac{\theta}{2}}\right)\cos\frac{\theta}{2} = \frac{\pi}{2} u \cot\frac{\theta}{2}$ //

6.



$|v_A| = 12 m/s$



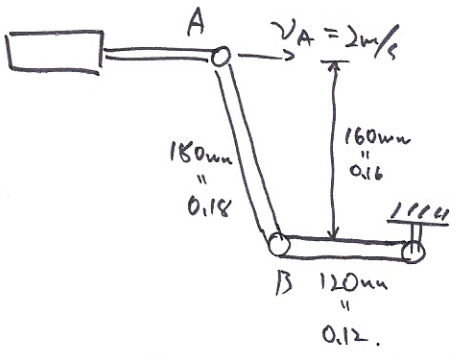
$$v_{A/O} = rw = r \cdot \frac{v_0}{r} = v_0$$

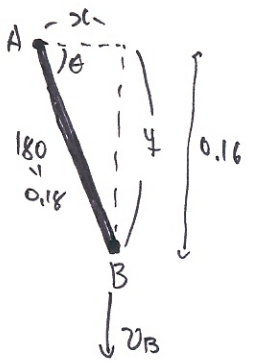
$$v_A = \sqrt{v_{A/O}^2 + v_0^2} = \sqrt{v_0^2 + v_0^2} = \sqrt{2} v_0 = 12$$

$$\therefore v_0 = \frac{12}{\sqrt{2}} = 8.49 m/s$$

$$w = \frac{v_0}{r} = \frac{8.49}{0.325} = 26.1 m/s.$$

7.





$x = 0.18\cos\theta$, $y = 0.18\sin\theta$

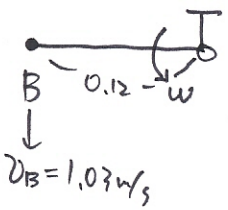
$\dot{x} = -2 = -0.18\dot{\theta}\sin\theta$ $\left(\sin\theta = \frac{0.16}{0.18} \Rightarrow \theta = 62.73^\circ\right)$

$\dot{\theta} = \frac{2}{0.18 \cdot \sin\theta}$ $(\cos\theta = \cos 62.73 = 0.46)$

$= \frac{2}{0.18 \cdot 0.89} = 12.48 \text{ rad/s}$

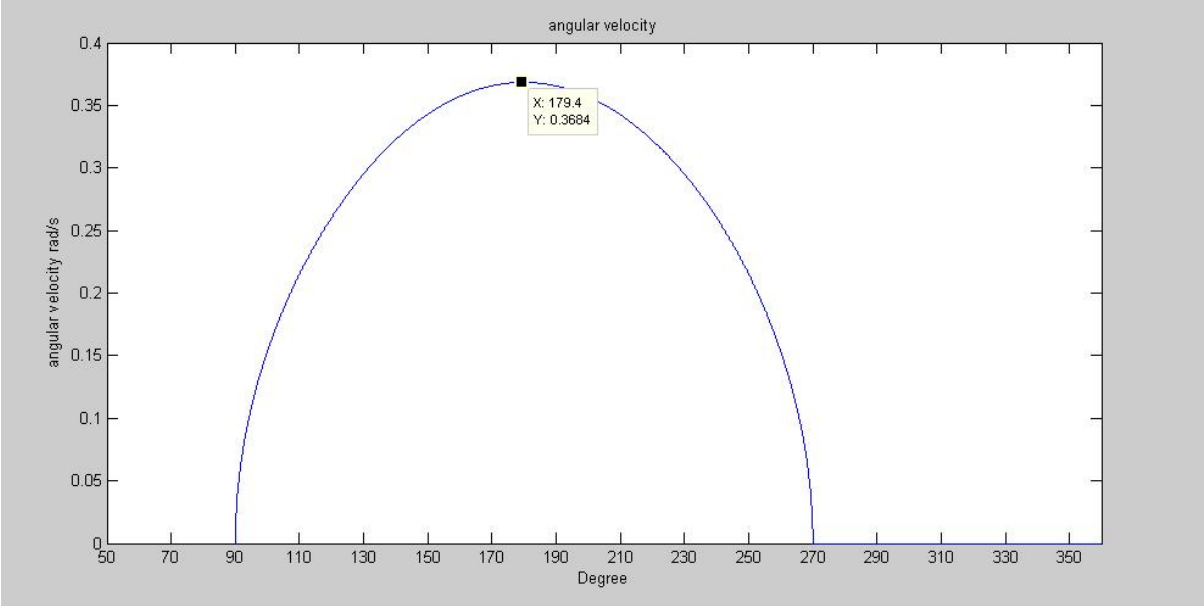
$v_B = \dot{y} = 0.18 \cdot \dot{\theta} \cdot \cos\theta$

$= 0.18 \cdot (12.48) \cdot 0.46 = 1.03 \text{ m/s}$



$\omega = \frac{v_B}{0.12} = \frac{1.03}{0.12} = \underline{8.58 \text{ rad/s}}$ //

13.C6



Sourse code

```
ang=pi/2:0.001:2*pi;                                    %호도법 각도 변수 생성
Degree=ang/pi*180;                                      %360분법 각도 변수 생성
Ls=(0.1^2+0.24^2+0.048.*cos(ang)).^(1/2);               %스프링 전체길이
DelLs=Ls-0.26;                                          %스프링 길이 변화량
w=((0.3.*DelLs.^2+9.8.*0.24.*sin(ang-pi/2)).*0.24^2).^(1/2);
                                                        %에너지법 이용 각속도식 생성
plot(Degree,w);                                         %각도,각속도 그래프그리기
title('angular velocity');
xlabel('Degree');
ylabel('angular velocity rad/s');                       %제목 및 labeling.
```