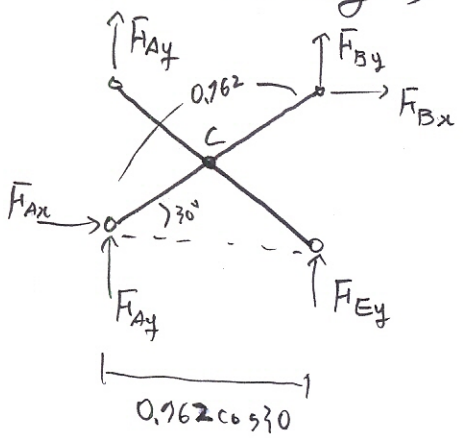
Hw #7 solution

1. FBD (linkage)

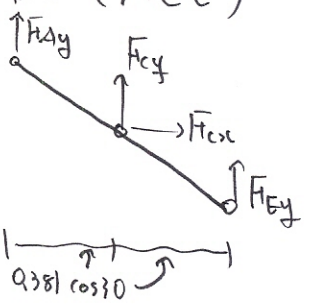


$$\circlearrowleft +\sum M_D = 0$$

$$(F_{By} + F_{Ey})(0.762\cos 30) - F_{Bx}(0.762\sin 30) = 0$$

$$(F_{By} + F_{Ey})\cos 30 - F_{Bx}\sin 30 = 0 \quad \cdots ①$$

FBD (ACE)



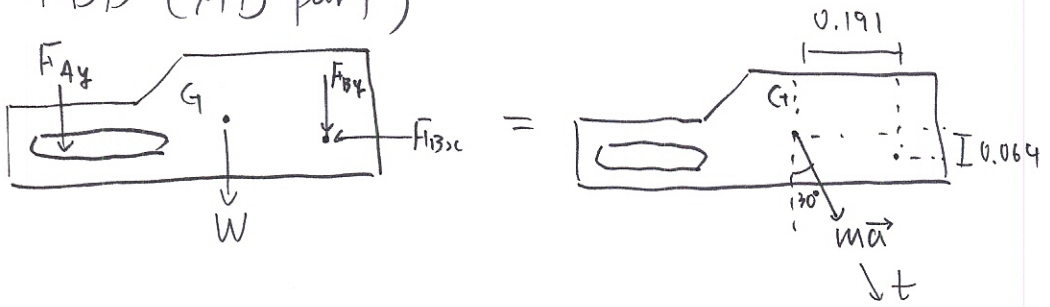
$$\circlearrowleft +\sum M_C = 0$$

$$-F_{Ay}(0.381\cos 30) + F_{Ey}(0.381\cos 30) = 0$$

$$F_{Ay} = F_{Ey} \quad \cdots ②$$

$$①, ② \rightarrow (F_{By} + F_{Ay})\cos 30 - F_{Bx}\sin 30 = 0 \quad \cdots ③$$

FBD (AB part)



(a) $\nearrow_{30}$ $\sum F_t = \sum F_{eff}$:

$$\underbrace{(F_{By} + F_{Ay})\cos 30 - F_{By}\sin 30}_{③ \,= 0} + W\cos 30 = ma$$

$$\therefore \quad a = \frac{W}{m}\cos 30 = g\cos 30 = \underline{8.5\ m/s^2 \nearrow_{30}}$$

(b) $\circlearrowleft + \Sigma M_B = \Sigma (M_B)_{eff}$

$$W(0.191) + F_{Ay}(0.762)\cos 30 = (m\bar{a}\sin 60)\,0.191 - ma(\cos 60)(0.064)$$

$$ma = W\cos 30 \quad \cdots (a)$$

$$0.191\,W + 0.762\,F_{Ay}\cos 30 = W\cos 30\,(0.19)\sin 60 - 0.064\cos 60)$$

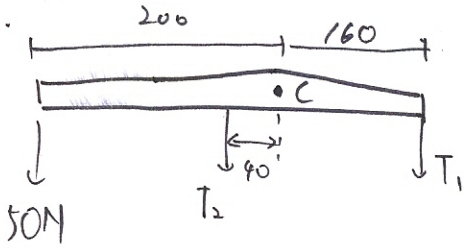
$$F_{Ay} = \frac{-0.191}{0.762\cos 30}W + \frac{W}{0.762}(0.191\sin 60 - 0.064\cos 60)$$

$(W = 88.3\,N)$

$$= -10.1\,N$$

$$\therefore \underline{F_{Ay} = 10.1\,N \uparrow}$$

2.



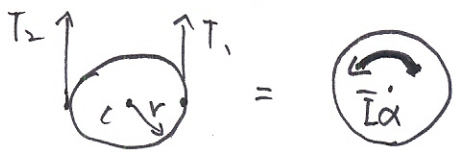
moment equillibrium

$\curvearrowleft \Sigma M_C = 0$

$$50(0.2) + T_2(0.04) - T_1(0.16) = 0$$

$$4T_1 - T_2 = 250 \quad \cdots \text{①}$$

FBD (wheel)



Eq of motion

$$\bar{I} = m k^2 = 300(0.6)^2$$

$$= 108 \text{ kg} \cdot \text{m}^2$$

$$\curvearrowleft \Sigma M_c = T_1 r - T_2 r = \bar{I}\alpha$$

$$T_1 - T_2 = \frac{108\alpha}{0.1}$$

$$\alpha = 925.9 \times 10^{-6}(T_1 - T_2) \quad \cdots\cdots \text{②}$$

o Belt friction

$$\frac{T_2}{T_1} = e^{\mu\theta} \qquad T_2 = T_1 e^{\mu\theta}$$

$\mu_k = 0.3$ $\theta = \pi$

$$T_2 = 2.663\, T_1 \quad \cdots\cdots \text{③}$$

D. ③ $4T_1 - T_2 = 250$

$4T_1 - 2,586T_1 = 250$

$T_1 = 174,38 \quad T_2 = 447,51N$

② → $\alpha = 925,9 \times 10^{-6}(174,38 - 447,51)$

$= -0,2529 \ rad/s^2$

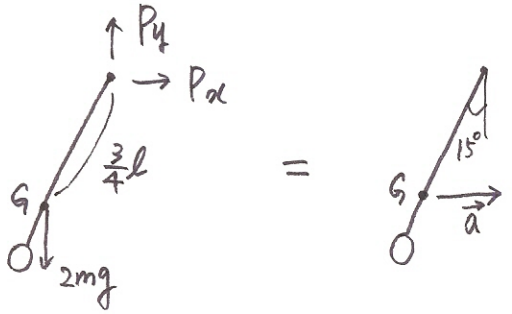
$\omega_0 = 180 \ rpm = 18,850 \ rad/s \ \circlearrowleft$

$\alpha = 0,2529 \ rad/s^2 \ \circlearrowright$

$\omega = \omega_0 + \alpha t \Rightarrow 0 = 18850 - 0,2529t$

$t = 74,5 \ s$ //

3.

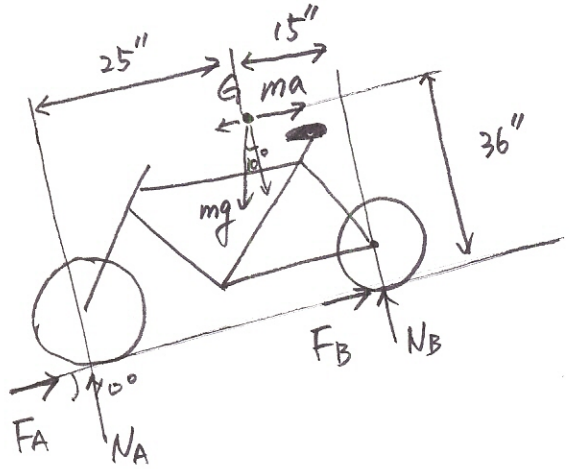


$$\circlearrowleft + \Sigma M_p = 2mg \sin 15° \times \frac{3}{4}\ell = 2ma \cos 15° \times \frac{3}{4}\ell$$

$$a = g \tan 15°$$

$$= 0.268\, g.$$

4.



Tipping at front wheel: N_B & $F_B \to 0$.

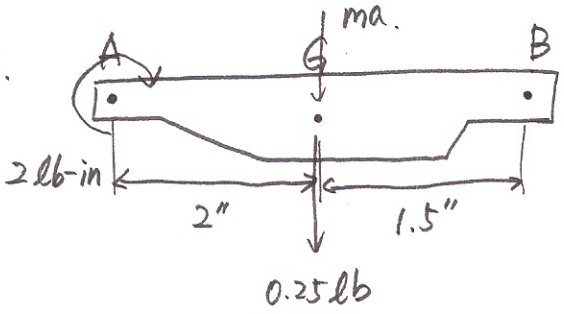
$$+\circlearrowright \Sigma M_A = ma\, d.$$

$$mg\,(25 \cos 15° - 36 \sin 10°) = ma \times 36$$

$$a = 0.51\, g$$

$$= 16.43 \text{ ft/sec}^2 \quad (\because g = 32.2 \text{ ft/sec}^2)$$

5.



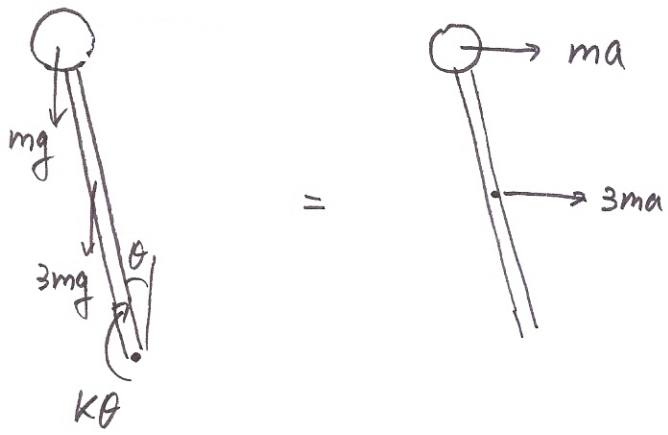
$$\Sigma M_A = ma\,d.$$

$$2 + 0.25 \times 2 = \frac{0.25}{32.2 \times 12} \times 2 \times a \qquad (\because 1\,ft = 12\,inch)$$

$$a = 1932\ inch/sec^2$$

$$= 161\ ft/sec^2$$

6. $a = 2g$ $\quad m = 0.5\,kg$ $\quad l = 0.6\,m$ $\quad \theta = 20°$

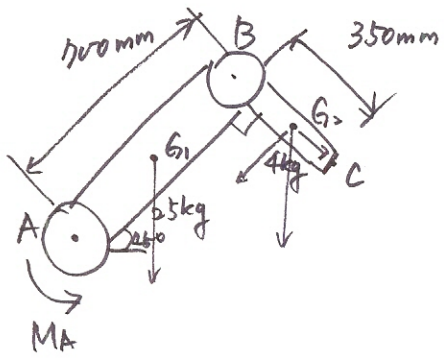


$\circlearrowleft^{+} \Sigma M_o \qquad k\theta - 3mg\sin\theta\frac{l}{2} - mg\sin\theta\, l = 3ma\cos\theta\frac{l}{2} + ma\cos\theta\, l$

$$k\theta - \frac{5}{2}mgl\sin\theta = \frac{5}{2}mal\cos\theta$$

$$\therefore K = 46.8\ N\cdot m/rad.$$

7.



$$I_B = \frac{1}{12} m_{BC} L_{BC}^2 + m_{BC} d^2 \quad \left(d = \frac{1}{2} L_{BC}\right)$$

$$= \frac{1}{3} m_{BC} L_{BC}^2$$

$$= \frac{1}{3} \times 4 \times 0.35^2$$

$$= 0.1633 \ kg \cdot m^2$$

$$I_A = \frac{1}{3} m_{AB} L_{AB}^2 + \frac{1}{12} m_{BC} L_{BC}^2 + m_{BC} \left\{ L_{AB}^2 + (L_{BC}/2)^2 \right\}$$

$$= \frac{1}{3} \times 25 \times 0.7^2 + \frac{1}{12} \times 4 \times 0.35^2 + 4(0.7^2 + 0.175^2)$$

$$= 6.21 \ kg \cdot m^2$$

$$\curvearrowright + \ \Sigma M_A = I_A \alpha$$

$$M_A - m_{AB} g \times \cos 45^\circ \times \frac{L_{AB}}{2} - m_{BC} g \cos 45^\circ \left(L_{AB} + \frac{L_{BC}}{2}\right)$$

$$= I_A \alpha$$

$$M_A - 25 \times 9.81 \times \cos 45^\circ \times 0.35 - 4 \times 9.81 \times \cos 45^\circ (0.7 + 0.175)$$

$$= 6.21 \times 4$$

$$\therefore M_A = 109.8 \ N \cdot m$$

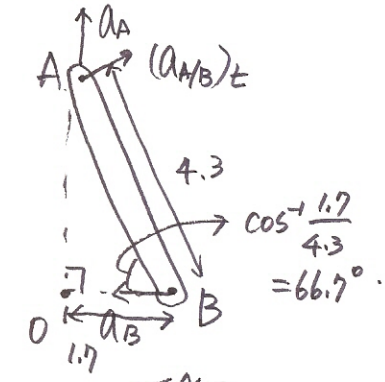
$$\curvearrowleft + \ \Sigma M_B = I_B \alpha$$

$$M_B - 4 \times 9.81 \times 0.175 \times \cos 45^\circ = 0.1633 \times 4$$

$$\therefore M_B = 5.51 \ N \cdot m$$

8. engine weight 1.2 lb.

radius of gyration about G 1.12 in.



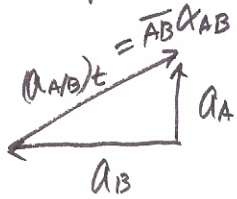
$$a_A = a_B + (a_{A/B})_t$$

$$(a_{A/B})_n = \overline{AB}\,\omega_{AB}^2 = 0$$

$$a_B = r\omega^2$$

$$= \frac{1.7}{12}(100\pi)^2$$

$$= 13.98 \times 10^3 \text{ ft/sec}^2.$$



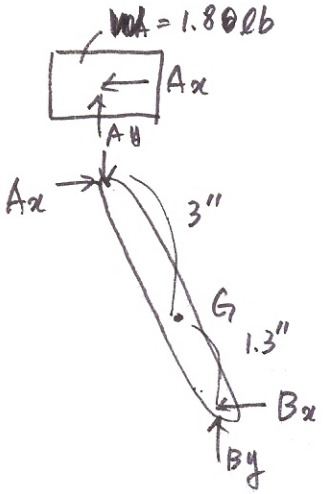
$$(a_{A/B})_t = \frac{a_B}{\sin 66.7°}$$

$$= 15.22 \times 10^3 \text{ ft/sec}^2$$

$$\therefore \alpha_{AB} = \frac{15.22 \times 10^3}{4.3/12} = 42.5 \times 10^3 \text{ rad/sec}^2 \circlearrowright$$

$$a_A = 13.98 \times 10^3 / \tan 66.7°$$

$$= 6.02 \times 10^3 \text{ ft/sec}^2$$



$$\Sigma F_y = ma_y \quad A_y = \frac{1.80}{32.2} \times 6.02 \times 10^3 = 336 \text{ lb}.$$

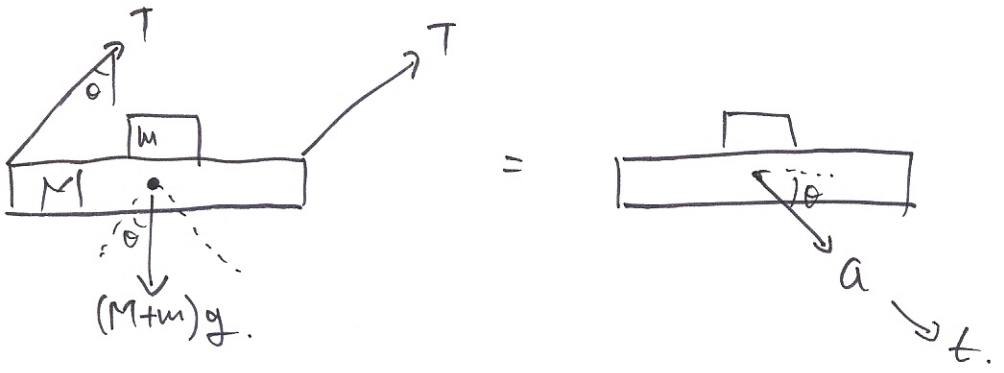
$$\Sigma M_B = I_B\alpha + \vec{r} \times m\vec{a}_B$$

$$336 \times 1.7 - A_x \times 3.95 = \frac{1.20}{32.2 \times 12}\left[1.12^2 + 1.3^2\right](-42.5)\times 10^3$$

$$+ 1.3 \times \frac{1.20}{32.2 \times 12}(13.98) \times 10^3 \times 12 \times \sin 66.7°$$

$$\therefore A_x = 85.6 \text{ lb} \qquad A = 347 \text{ lb}.$$

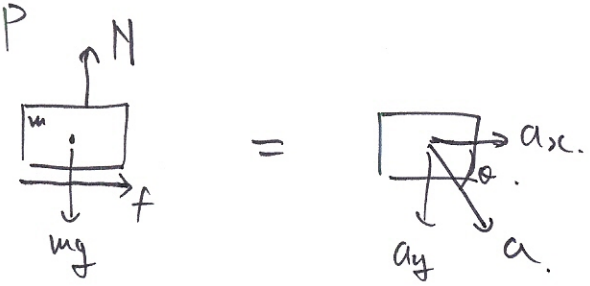
9.



ΣF_t : $(M+m)g\sin\theta = (M+m)a$

$a = g\sin\theta \qquad a_y = g\sin^2\theta \qquad a_x = g\sin\theta\cos\theta.$

no slip



ΣF_y : $N - mg = -ma_y.$

$N = m(g-a_y) = mg(1-\sin^2\theta) = mg\cos^2\theta$

ΣF_x : $f = ma_x.$

$\mu N = ma_x$

$\mu mg\cos^2\theta = mg\sin\theta\cos\theta.$

$\mu = \dfrac{\sin\theta}{\cos\theta}$

```
ang=0:0.01:40*pi/180;
mu = sin(ang)./cos(ang);
deg=ang*180/pi;
plot(deg,mu);
xlabel('ang'); ylabel('friction coef.');
```