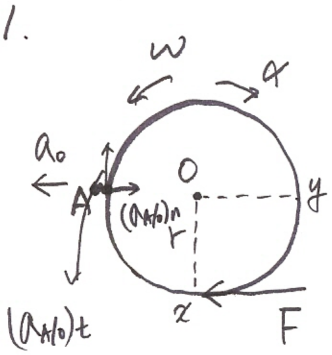
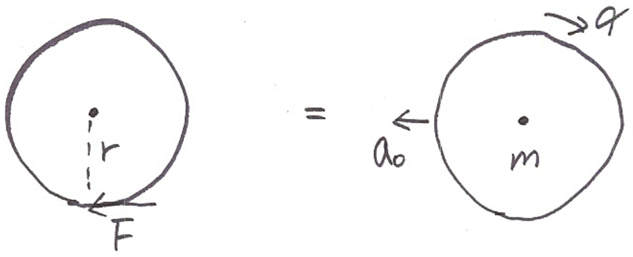
Homework #8 solution.

1.





$\Sigma M_o = I_o\alpha \qquad F\cdot r = mk^2\alpha \qquad \alpha = \dfrac{Fr}{mk^2}$

$\Sigma F_y = m a_y \qquad -F = m(-a_o)$

$a_o = F/m \quad \leftarrow \left(-F/m\, j\right)$

$\bar{a}_A = \bar{a}_o + \bar{a}_{A/o}$

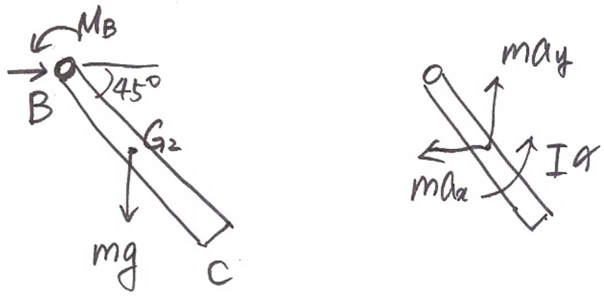
$(a_{A/o})_n = r\omega^2 \quad \rightarrow \quad (r\omega^2 j)$

$(a_{A/o})_t = -r\alpha$

$= -\dfrac{Fr^2}{mk^2} \quad \uparrow \quad \left(-\dfrac{Fr^2}{mk^2} i\right)$

$\bar{a}_A = -\dfrac{Fr^2}{mk^2} i - \left(\dfrac{F}{m} - r\omega^2\right) j$

2.



$$\omega_{BC} = \omega_{AB} = 2\hat{k} \text{ rad/s}$$

$$\alpha_{BC} = \alpha_{AB} = 4\hat{k} \text{ rad/s}^2$$

$$\vec{a}_{G_2} = \bar{\alpha} \times \bar{r}_{AG_2} - \omega^2 \bar{r}_{AG_2}$$

$$= 4\hat{k} \times \left[(0.7 + 0.175)\cos 45^\circ \hat{i} + (0.7 - 0.175)\sin 45^\circ \hat{j} \right]$$

$$- 2^2 \left[(0.7 + 0.175)\cos 45^\circ \hat{i} + (0.7 - 0.175)\sin 45^\circ \hat{j} \right]$$

$$= -3.96\hat{i} + 0.990\hat{j} \text{ m/s}^2$$

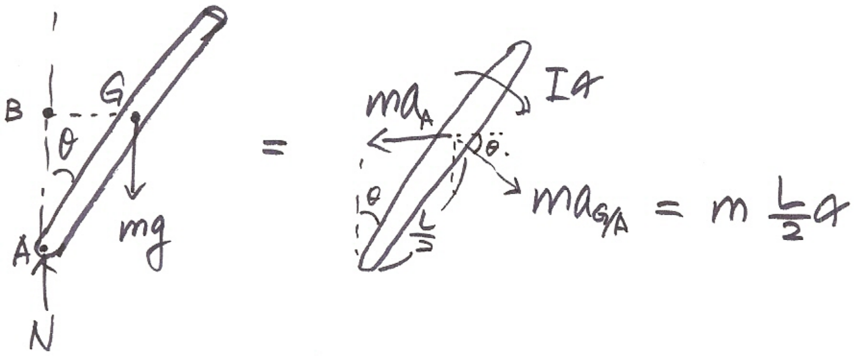
$$\Sigma M_B = I\alpha + mad$$

$$M_B - mg \times 0.175 \sin 45^\circ = \frac{1}{12} \times 4 \times 0.35^2 \times 4 + 4 \times 0.990 \times 0.175 \cos 45^\circ$$

$$- 4 \times 3.96 \times 0.175 \times \sin 45^\circ$$

$$M_B = 3.55 \text{ N}\cdot\text{m} \circlearrowleft$$

3.



$$\circlearrowleft \Sigma M_B = I\alpha + \Sigma ma\,d.$$

$$mg\frac{L}{2}\sin\theta = \frac{1}{12}mL^2\alpha + \frac{L}{2}\sin^2\theta\left(m\frac{L}{2}\alpha\right)$$

$$\therefore\ \alpha = \frac{2g}{L}\frac{\sin\theta}{1/3 + \sin^2\theta}$$

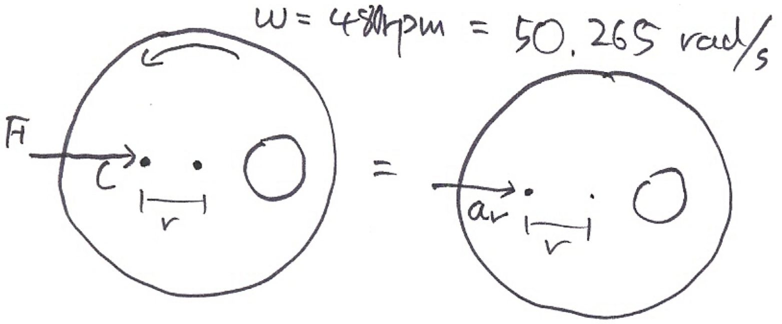
$$\Sigma F_y: \quad mg - N = m\cdot\frac{L}{2}\alpha\sin\theta$$

$$N = \frac{mg}{1 + 3\sin^2\theta}$$

4.

FBD

$\omega = 480\,rpm = 50.265\ rad/s$



Force (r-direction)

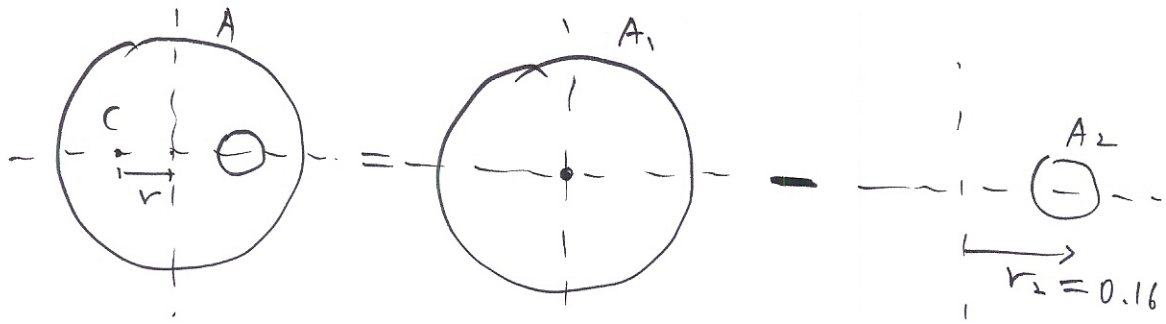
$$\Sigma F_r = -F = ma_r$$

$$-F = m(\ddot{r} - r\dot{\theta}^2)$$

$\ddot{r} = 0$

$$= -mr\dot{\theta}^2 \Rightarrow F = mr\omega^2 \cdots (1)$$

∘ center of mass



$$\bar{r}A = \bar{r}_1 A_1 - r_2 A_2$$

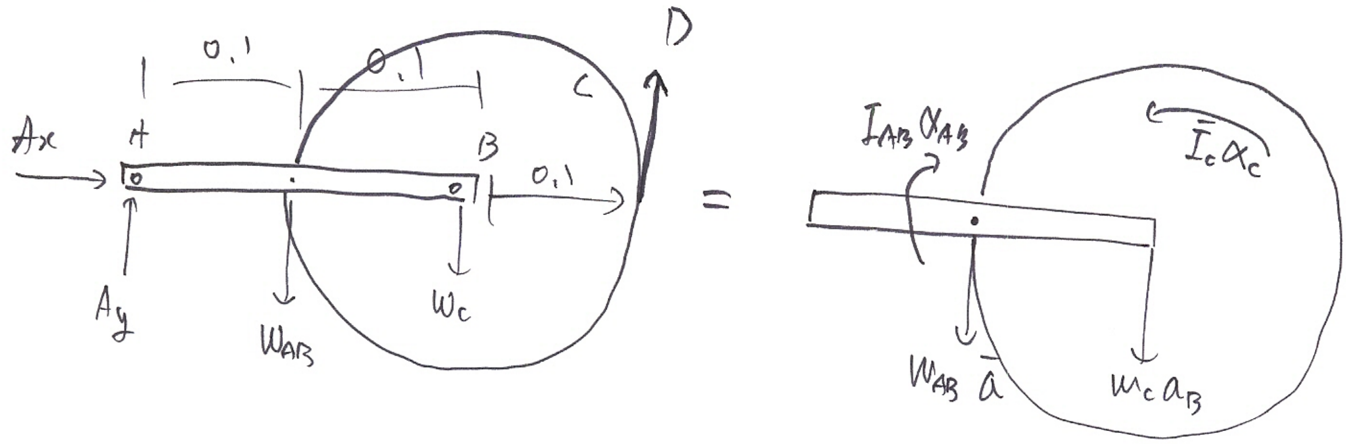
$\bar{r}_1 = 0$

$$\bar{r} = \frac{0 - 0.16\cdot\pi(0.06)^2}{A_1 - A_2} = \frac{-0.16\pi(0.06)^2}{\pi(0.3)^2 - \pi(0.06)^2} = -0.00667\ m$$

$\therefore$ (1) → $F = m\cdot r\cdot\omega^2 = (30)(6.667\times10^{-3}\ m)(50.265)$

$= \underline{\underline{505\ N}} \rightarrow$

5. FBD

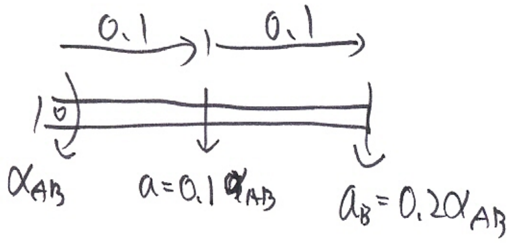


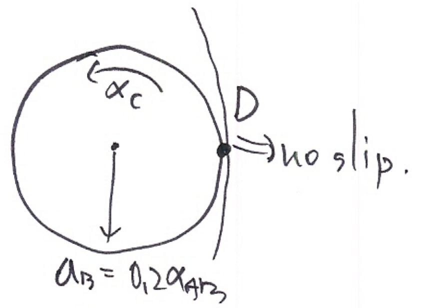
$$+\circlearrowleft \sum M_A = W_{AB}(0.1) + W_c(0.2) - D(0.3) = (m_{AB}a)0.1 + (m_c a_B)0.2 + I_{AB}\alpha_{AB} - \bar{I}_c\alpha_c$$

$$3g(0.1) + 5g(0.2) - D(0.3) = 3\cdot\bar{a}\cdot 0.1 + 5a_B\cdot 0.2 + \frac{1}{12}(3)(0.2)^2\alpha_{AB} - 5(0.075)^2\alpha_c$$

- - - - - - - - (1)

Kinematics



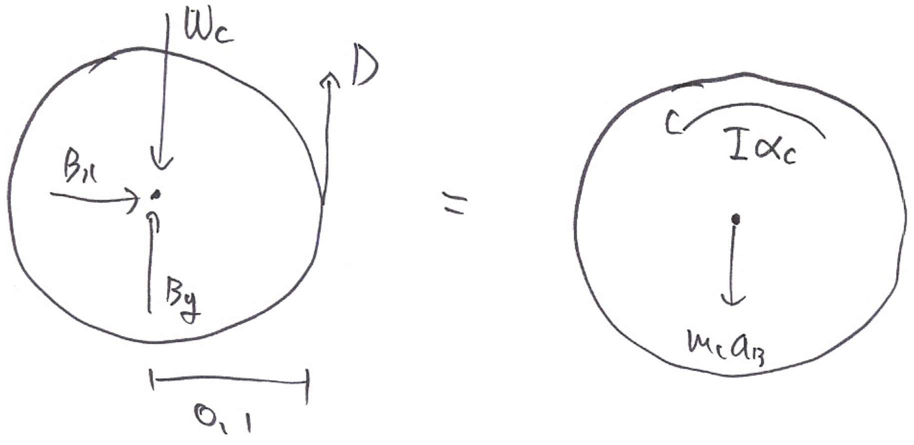


⇓

acceleration $D\overset{+}{\uparrow}$: $0 = \alpha_c(0.1) - 0.2\alpha_{AB}$

$$\alpha_{AB} = \frac{1}{2}\alpha_c \quad \cdots \quad (2)$$

gear C



$$\circlearrowleft + \; \Sigma M_B : \quad D(0.1) = I_c \alpha_c$$

$$= (5)(0.075)^2 \alpha_c$$

$$D = 0.28125 \alpha_c \quad \cdots (3)$$

(a) (1), (2), (3)

$$\Rightarrow \quad (1.3)g - 0.3(0.28125\alpha_c) = 0.24\, a_{AB} - 0.028125\alpha_c$$

$$(1.3)g - 0.3(0.28125\alpha_c) = 0.24\left(\tfrac{1}{2}\alpha_c\right) - 0.28125\alpha_c$$

$$1.3g = 0.17625\alpha_c \qquad \alpha_c = 7.3759g$$

$$= \underline{\underline{72.36\ (rad/s^2)\circlearrowleft}}$$

(b) $a_B = 0.2\alpha_{AB} = 0.2\left(\tfrac{1}{2}\alpha_c\right) = 0.1\alpha_c = 0.1(72.36)$

$$= \underline{\underline{7.24\ m/s^2 \downarrow}}$$