1. Derive the Laplace transforms of the exponentially decaying sine and cosine functions, $e^{-at}\sin\omega t$ and $e^{-at}\cos\omega t$, where a and ω are constants.

2. Derive the Laplace transform of the function $t\cos\omega t$.

3. Determine the complete response of the following model, which has a ramp input:

$$\dot{x}+3x=5t \qquad x(0)=10$$

4. Use two methods to obtain the inverse Laplace transform of

$$X(s)=\frac{3s+7}{4s^2+24s+136}=\frac{3s+7}{4(s^2+6s+34)}$$

5. Use the Laplace transform to solve the following problem:

$$\ddot{x}+6\dot{x}+34x=5\sin 6t \qquad x(0)=0 \qquad \dot{x}(0)=0$$

6. (a) Derive the Laplace transform of the function $A\sin(\omega t+\phi)$.
   (b) Generalize the answer from part (a) to find the Laplace transform of $Ae^{-at}\sin(\omega t+\phi)$.

7. Obtain the inverse Laplace transform of

$$X(s)=\frac{1-e^{-2s}}{s(s+4)}$$

8. Suppose a rectangular pulse P(t) of unit height and duration 2 is applied to the first-order model $\dot{x}+4x=P(t)$ with a zero initial condition. Use the Laplace transform to determine the response. (***Draw a graph of the response.***)

9. Obtain the transfer functions X(s)/F(s) and Y(s)/F(s) for the following model:

$$3\dot{x}=y \qquad \dot{y}=f(t)-3y-15x$$

10. Solve the following Problems for x(t). Compare the values of $x(0+)$ and $x(0)$.
    For parts (b) through (d), also compare the values of $\dot{x}(0+)$ and $\dot{x}(0)$.

a. $7\dot{x}+5x=4\delta(t) \qquad x(0)=3$

b. $\ddot{x}+14\dot{x}+49x=3\delta(t) \qquad x(0)=2 \qquad \dot{x}(0)=3$

11. Obtain the Laplace transform of the function plotted in Figure 1.

12. A trapezoidal profile f(t) is shown in Figure 2. Obtain the response x(t) of the following model:

$$2\dot{x}+x=f(t) \qquad x(0)=0$$

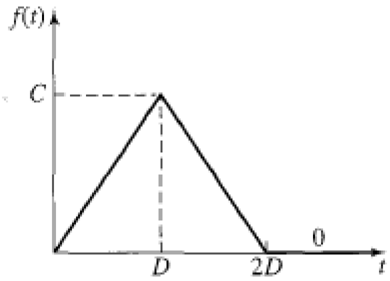

Fig 1.

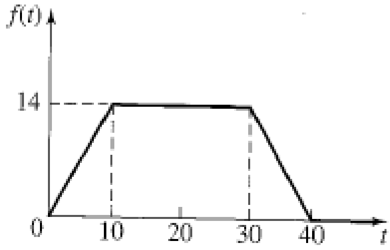

Fig 2.