1. 단일 에너지 장벽에 대한 투과계수의 그래프 (강의자료 #6의 슬라이드 #3)를 컴퓨터를 이용하여 그리시오 (E<U, E>U 의 경우 모두 포함).

E<U 인 경우와 E>U 인 경우 모두 파동방정식은 다음과 같은 꼴이 된다.

$$\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$$

i) $x < 0$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + U\psi(x) = E\psi(x)$$

$$\psi(x) = Ae^{ik_1x} + Be^{-ik_1x} \quad \text{where } k_1 = \frac{\sqrt{2mE}}{\hbar}$$

ii) $0 < x < L$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) = E\psi(x)$$

$$\psi(x,t) = Ce^{ik_2x} + De^{-ik_2x} \quad \text{where } k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$$ (U>E 일 경우 허수)

iii) $x > L$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + U\psi(x) = E\psi(x)$$

$$\psi(x) = Fe^{ik_1x} + Ge^{-ik_1x} \quad \text{where } k_1 = \frac{\sqrt{2mE}}{\hbar}$$

입자가 왼쪽에서 입사되기 때문에 G = 0

Boundary Condition

$\psi(0)$: $A + B = C + D$ (1)

$\psi'(0)$: $k_1A - k_1B = k_2C - k_2D$ (2)

$\psi(L)$: $Ce^{ik_2L} + De^{ik_2L} = Fe^{ik_1L}$ (3)

$\psi'(L)$: $-k_2Ce^{ik_2L} + k_2De^{ik_2L} = k_1Fe^{ik_1L}$ (4)

(1)과 (2) 연립 : $C = \frac{k_1+k_2}{2k_2}A + \frac{k_2-k_1}{2k_2}B$

$D = \frac{k_2-k_1}{2k_2}A + \frac{k_1+k_2}{2k_2}B$

(3)과 (4) 연립 : $C = \frac{k_1+k_2}{2k_2}Fe^{\frac{k_1-k_2}{2}L}$

$D = \frac{k_2-k_1}{2k_2}Fe^{\frac{k_1+k_2}{2}L}$

C, D를 소거하고, F를 A에 관하여 나타내면,

$$F = A\frac{4k_1k_2}{(k_1+k_2)^2 e^{i(k_1-k_2)L} - (k_2-k_1)^2 e^{i(k_1+k_2)L}}$$

$$F^* F = A^* A\frac{16k_1^2k_2^2}{(k_1+k_2)^4 + (k_2-k_1)^4 - (k_1+k_2)^2(k_2-k_1)^2(e^{2ik_2L}+e^{-2ik_2L})}$$

$$T = \frac{F^* F}{A^* A} = \frac{16k_1^2k_2^2}{2k_1^4+2k_2^4+12k_1^2k_2^2-(k_1^2-k_2^2)^2(e^{2ik_2L}+e^{-2ik_2L})}$$

$$= \frac{16k_1^2k_2^2}{2k_1^4+2k_2^4+12k_1^2k_2^2-(k_1^2-k_2^2)^2(2-4\sin^2 k_2L)}$$

$$= \frac{16k_1^2k_2^2}{16k_1^2k_2^2+(k_1^2-k_2^2)^2(4\sin^2 k_2L)}$$

$$= \frac{1}{1+\frac{(k_1^2-k_2^2)^2}{k_1^2k_2^2}\frac{1}{4}sin^2 k_2L}$$

$$= \frac{1}{1+\frac{U^2}{4E(E-U)}sin^2 k_2L}$$

$E \geq U$ 일 경우 k_2는 실수.

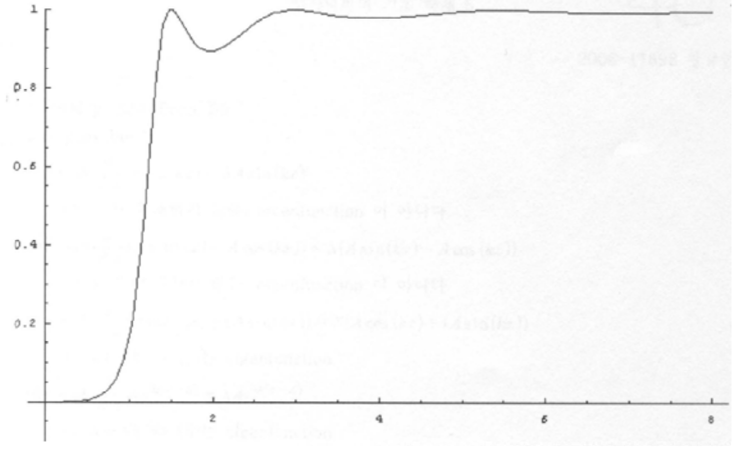
따라서 $T = \left[1+\frac{U^2}{4E(E-U)}sin^2 k_2L\right]^{-1}$, where $k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$

$E < U$ 일 경우 k_2는 허수.

$k_2 = i\alpha$ 를 대입하면,

따라서 $T = \left[1+\frac{U^2}{4E(U-E)}sinh^2 \alpha L\right]^{-1}$ where $\alpha = \frac{k_2}{i} = \frac{\sqrt{2m(U-E)}}{\hbar}$

Mathematica 5.2 를 이용한 plot 결과 (세로축 T, 가로축 E/U)



2. 교재 p. 199, Exercise #37.

5-37: (a) In the notation of the Appendix, the wave function in the two regions has the form

$$\psi_I = A\,e^{i k_1 x} + B\,e^{-i k_1 x}, \qquad \psi_{II} = C\,e^{i k' x} + D\,e^{-i k' x},$$

where
$$k_1 = \sqrt{\frac{2mE}{\hbar}}, \qquad k' = \sqrt{\frac{2m(E-U)}{\hbar}}.$$

The terms corresponding to $e^{-i k_1 x}$ and $e^{-i k' x}$ represent particles traveling to the left; this is possible in region I, due to reflection at the step at $x = 0$, but not in region II (the reasoning is the same as that which lead to setting $G = 0$ in Equation (5.82)). Therefore, the $e^{-i k' x}$ term is not physically meaningful, and $D = 0$.

(b) The boundary conditions at $x = 0$ are then

$$A + B = C, \qquad i k_1 A - i k_1 B = i k' C \qquad \text{or} \qquad A - B = \frac{k'}{k_1} C.$$

Adding to eliminate B, $2A = \left(1 + \frac{k'}{k_1}\right) C$, so

$$\frac{C}{A} = \frac{2k_1}{k_1 + k'}, \qquad \text{and} \qquad \frac{CC^*}{AA^*} = \frac{4k_1^2}{(k_1 + k')^2}.$$

(Note that the ratios C/A and C^*/A^* are real in this case.)

(c) The particle speeds are different in the two regions, so Equation (5.83) becomes

$$T = \frac{|\psi_{II}|^2\, v'}{|\psi_I|^2\, v_1} = \frac{CC^*}{AA^*}\frac{k'}{k_1} = \frac{4k_1 k'}{(k_1 + k')^2} = \frac{4\,(k_1/k')}{((k_1/k') + 1)^2}.$$

For the given situation, $k_1/k' = v_1/v' = 2.00$, so $T = \frac{(4)(2)}{((2)+1)^2} = \frac{8}{9}$. The transmitted current is $(T)(1.00\text{ mA}) = 0.889$ mA, and the reflected current is 0.111 mA.

As a check on the last result, note that the ratio of the reflected current to the incident current is, in the notation of the Appendix,

$$R = \frac{|\psi_{I+}|^2\, v_1}{|\psi_{I+}|^2\, v_1} = \frac{BB^*}{AA^*}.$$

Eliminating C from the equations obtained in part (b) from the continuity condition as $x = 0$,

$$A\left(1 - \frac{k'}{k_1}\right) = B\left(1 + \frac{k'}{k_1}\right), \qquad \text{so}$$

$$R = \left(\frac{(k1/k') - 1}{(k_1/k') + 1}\right)^2 = \frac{1}{9} = 1 - T,$$

as expected.

3. 교재 p. 189의 식 (5.69)에서 출발하여 식 (5.70)을 증명하시오.

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#3. $\frac{d^2\psi}{dy^2} + (\alpha - y^2)\psi = 0$

let $\psi = \left(\sum_{k=0}^{\infty} a_k y^k\right) e^{-\frac{y^2}{2}}$

$\sum_{k=0}^{\infty} a_k y^k \to h(y)$

$\frac{d\psi}{dy} = \left(\frac{dh}{dy} - yh\right)e^{-\frac{y^2}{2}}$

$\frac{d^2\psi}{dy^2} = \left(\frac{d^2h}{dy^2} - 2y\frac{dh}{dy} + (y^2-1)h\right)e^{-\frac{y^2}{2}}$

Schrödinger equation

$\Rightarrow \boxed{\frac{d^2h}{dy^2} - 2y\frac{dh}{dy} + (\alpha - 1)h = 0 \cdots ①}$

$\frac{dh}{dy} = a_1 + 2a_2 y + 3a_3 y^2 + \cdots = \sum_{k=0}^{\infty} k a_k y^{k-1}$

$\frac{d^2h}{dy^2} = 2a_2 + 2\cdot 3 a_3 y + 3\cdot 4 a_4 y^2 + \cdots = \sum_{k=0}^{\infty} (k+1)(k+2)a_{k+2} y^k$

Putting these into eq ①,

$\sum_{k=0}^{\infty}\left[(k+1)(k+2)a_{k+2} - 2ka_k + (\alpha-1)a_k\right]y^k = 0$

$\Rightarrow (k+1)(k+2)a_{k+2} - 2ka_k + (\alpha-1)a_k = 0$

$\therefore a_{k+2} = \frac{(2k+1-\alpha)}{(k+1)(k+2)} a_k \cdots ②$

There must occur some "highest" k (call it n)

such that the formula ② spits out $a_{n+2} = 0$

(for normalizable solutions)

$\therefore \alpha = 2n+1 \quad (n = 0, 1, 2, \cdots)$

$= \frac{E}{h\nu}$

$\Rightarrow \boxed{E = \frac{h\nu}{2}(2n+1) \quad (n = 0, 1, 2, \ldots)}$

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