Chapter 1 Answers

1.1. Converting from polar to Cartesian coordinates:

$\frac{1}{2}e^{j\pi} = \frac{1}{2}\cos\pi = -\frac{1}{2}$, $\quad \frac{1}{2}e^{-j\pi} = \frac{1}{2}\cos(-\pi) = -\frac{1}{2}$

$e^{j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right) = j$, $\quad e^{-j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) = -j$

$e^{j5\frac{\pi}{2}} = e^{j\frac{\pi}{2}} = j$, $\quad \sqrt{2}e^{j\frac{\pi}{4}} = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right) = 1+j$

$\sqrt{2}e^{\frac{9j\pi}{4}} = \sqrt{2}e^{\frac{j\pi}{4}} = 1+j$, $\quad \sqrt{2}e^{\frac{-9j\pi}{4}} = \sqrt{2}e^{\frac{-j\pi}{4}} = 1-j$

$\sqrt{2}e^{\frac{-j\pi}{4}} = 1-j$

1.2. Converting from Cartesian to polar coordinates:

$5 = 5e^{j0}$, $\quad -2 = 2e^{j\pi}$, $\quad -3j = 3e^{-j\frac{\pi}{2}}$

$\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{-j\frac{\pi}{3}}$, $\quad 1+j = \sqrt{2}e^{j\frac{\pi}{4}}$, $\quad (1-j)^2 = 2e^{-j\frac{\pi}{2}}$

$j(1-j) = e^{j\frac{\pi}{4}}$, $\quad \frac{1+j}{1-j} = e^{j\frac{\pi}{2}}$, $\quad \frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}} = e^{-j\frac{\pi}{12}}$

1.3. (a) $E_\infty = \int_0^\infty e^{-4t}dt = \frac{1}{4}$, $P_\infty = 0$, because $E_\infty < \infty$

(b) $x_2(t) = e^{j(2t+\frac{\pi}{4})}$, $|x_2(t)| = 1$. Therefore, $E_\infty = \int_{-\infty}^{\infty}|x_2(t)|^2dt = \int_{-\infty}^{\infty}dt = \infty$, $P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x_2(t)|^2dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt = \lim_{T\to\infty}1 = 1$

(c) $x_3(t) = \cos(t)$. Therefore, $E_\infty = \int_{-\infty}^{\infty}|x_3(t)|^2dt = \int_{-\infty}^{\infty}\cos^2(t)dt = \infty$,

$$P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\cos^2(t)dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left(\frac{1+\cos(2t)}{2}\right)dt = \frac{1}{2}$$

(d) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$, $|x_1[n]|^2 = \left(\frac{1}{4}\right)^n u[n]$. Therefore, $E_\infty = \sum_{n=-\infty}^{\infty}|x_1[n]|^2 = \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n = \frac{4}{3}$.

$P_\infty = 0$, because $E_\infty < \infty$.

(e) $x_2[n] = e^{j(\frac{\pi n}{2}+\frac{\pi}{8})}$, $|x_2[n]|^2 = 1$. Therefore, $E_\infty = \sum_{n=-\infty}^{\infty}|x_2[n]|^2 = \infty$,

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x_2[n]|^2 = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}1 = 1.$$

(f) $x_3[n] = \cos(\frac{\pi}{4}n)$. Therefore, $E_\infty = \sum_{n=-\infty}^{\infty}|x_3[n]|^2 = \sum_{n=-\infty}^{\infty}\cos^2(\frac{\pi}{4}n) = \infty$,

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}\cos^2(\frac{\pi}{4}n) = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}\left(\frac{1+\cos(\frac{\pi}{2}n)}{2}\right) = \frac{1}{2}$$

1.4. (a) The signal $x[n]$ is shifted by 3 to the right. The shifted signal will be zero for $n < 1$ and $n > 7$.

(b) The signal $x[n]$ is shifted by 4 to the left. The shifted signal will be zero for $n < -6$ and $n > 0$.

(c) The signal $x[n]$ is flipped. The flipped signal will be zero for $n < -4$ and $n > 2$.

(d) The signal $x[n]$ is flipped and the flipped signal is shifted by 2 to the right. This new signal will be zero for $n < -2$ and $n > 4$.

(e) The signal $x[n]$ is flipped and the flipped signal is shifted by 2 to the left. This new signal will be zero for $n < -6$ and $n > 0$.

1.5. (a) $x(1-t)$ is obtained by flipping $x(t)$ and shifting the flipped signal by 1 to the right. Therefore, $x(1-t)$ will be zero for $t > -2$.

(b) From (a), we know that $x(1-t)$ is zero for $t > -2$. Similarly, $x(2-t)$ is zero for $t > -1$. Therefore, $x(1-t) + x(2-t)$ will be zero for $t > -2$.

(c) $x(3t)$ is obtained by linearly compressing $x(t)$ by a factor of 3. Therefore, $x(3t)$ will be zero for $t < 1$.

(d) $x(t/3)$ is obtained by linearly stretching $x(t)$ by a factor of 3. Therefore, $x(t/3)$ will be zero for $t < 9$.

1.6. (a) $x_1(t)$ is not periodic because it is zero for $t < 0$.

(b) $x_2[n] = 1$ for all n. Therefore, it is periodic with a fundamental period of 1.

(c) $x_3[n]$ is as shown in the Figure S1.6.

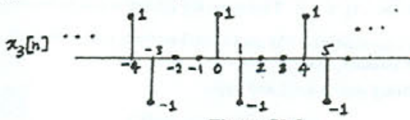

Figure S1.6

Therefore, it is periodic with a fundamental period of 4.

1.7. (a)

$$\mathcal{E}v\{x_1[n]\} = \frac{1}{2}(x_1[n] + x_1[-n]) = \frac{1}{2}(u[n] - u[n-4] + u[-n] - u[-n-4])$$

Therefore, $\mathcal{E}v\{x_1[n]\}$ is zero for $|n| > 3$.

(b) Since $x_2(t)$ is an odd signal, $\mathcal{E}v\{x_2(t)\}$ is zero for all values of t.

(c)

$$\mathcal{E}v\{x_3[n]\} = \frac{1}{2}(x_1[n] + x_1[-n]) = \frac{1}{2}[(\frac{1}{2})^n u[n-3] - (\frac{1}{2})^{-n}u[-n-3]]$$

Therefore, $\mathcal{E}v\{x_3[n]\}$ is zero when $|n| < 3$ and when $|n| \to \infty$.

(d)

$$\mathcal{E}v\{x_4(t)\} = \frac{1}{2}(x_4(t) + x_4(-t)) = \frac{1}{2}[e^{-5t}u(t+2) - e^{5t}u(-t+2)]$$

Therefore, $\mathcal{E}v\{x_4(t)\}$ is zero only when $|t| \to \infty$.

1.8. (a) $\mathcal{R}e\{x_1(t)\} = -2 = 2e^{0t}\cos(0t+\pi)$

(b) $\mathcal{R}e\{x_2(t)\} = \sqrt{2}\cos(\frac{\pi}{4})\cos(3t+2\pi) = \cos(3t) = e^{0t}\cos(3t+0)$

(c) $\mathcal{R}e\{x_3(t)\} = e^{-t}\sin(3t+\pi) = e^{-t}\cos(3t+\frac{\pi}{2})$

(d) $\mathcal{R}e\{x_4(t)\} = -e^{-2t}\sin(100t) = e^{-2t}\sin(100t+\pi) = e^{-2t}\cos(100t+\frac{\pi}{2})$

1.9. (a) $x_1(t)$ is a periodic complex exponential.

$$x_1(t) = je^{j10t} = e^{j(10t+\frac{\pi}{2})}$$

The fundamental period of $x_1(t)$ is $\frac{2\pi}{10} = \frac{\pi}{5}$.

(b) $x_2(t)$ is a complex exponential multiplied by a decaying exponential. Therefore, $x_2(t)$ is not periodic.

(c) $x_3[n]$ is a periodic signal.

$$x_3[n] = e^{j7\pi n} = e^{j\pi n}$$

$x_3[n]$ is a complex exponential with a fundamental period of $\frac{2\pi}{\pi} = 2$.

(d) $x_4[n]$ is a periodic signal. The fundamental period is given by $N = m(\frac{2\pi}{3\pi/5}) = m(\frac{10}{3})$. By choosing $m = 3$, we obtain the fundamental period to be 10.

(e) $x_5[n]$ is not periodic. $x_5[n]$ is a complex exponential with $\omega_0 = 3/5$. We cannot find any integer m such that $m(\frac{2\pi}{\omega_0})$ is also an integer. Therefore, $x_5[n]$ is not periodic.

1.10.

$$x(t) = 2\cos(10t+1) - \sin(4t-1)$$

Period of first term in RHS $= \frac{2\pi}{10} = \frac{\pi}{5}$

Period of second term in RHS $= \frac{2\pi}{4} = \frac{\pi}{2}$

Therefore, the overall signal is periodic with a period which is the least common multiple of the periods of the first and second terms. This is equal to π.

1.11.

$$x[n] = 1 + e^{j\frac{4\pi}{7}n} - e^{j\frac{2\pi}{5}n}$$

Period of the first term in the RHS = 1

Period of the second term in the RHS $= m(\frac{2\pi}{4\pi/7}) = 7$ (when $m = 2$)

Period of the third term in the RHS $= m(\frac{2\pi}{2\pi/5}) = 5$ (when $m = 1$)

Therefore, the overall signal $x[n]$ is periodic with a period which is the least common multiple of the periods of the three terms in $x[n]$. This is equal to 35.

1.12. The signal $x[n]$ is as shown in Figure S1.12. $x[n]$ can be obtained by flipping $u[n]$ and then shifting the flipped signal by 3 to the right. Therefore, $x[n] = u[-n+3]$. This implies that $M = -1$ and $n_0 = -3$.

Figure S1.12

1.13.

$$y(t) = \int_{-\infty}^{t}x(\tau)d\tau = \int_{-\infty}^{t}(\delta(\tau+2) - \delta(\tau-2))d\tau = \begin{cases} 0, & t < -2 \\ 1, & -2 \le t \le 2 \\ 0, & t > 2 \end{cases}$$

Therefore,

$$E_\infty = \int_{-2}^{2}dt = 4$$

1.14. The signal $x(t)$ and its derivative $g(t)$ are shown in Figure S1.14.

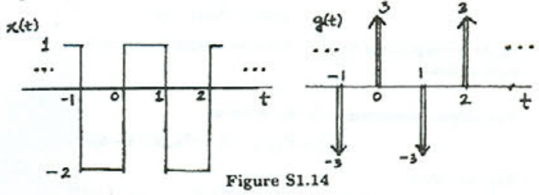

Figure S1.14

Therefore,

$$g(t) = 3\sum_{k=-\infty}^{\infty}\delta(t-2k) - 3\sum_{k=-\infty}^{\infty}\delta(t-2k-1)$$

This implies that $A_1 = 3$, $t_1 = 0$, $A_2 = -3$, and $t_2 = 1$.

1.15. (a) The signal $x_2[n]$, which is the input to S_2, is the same as $y_1[n]$. Therefore,

$$\begin{aligned} y_2[n] &= x_2[n-2] + \frac{1}{2}x_2[n-3] \\ &= y_1[n-2] + \frac{1}{2}y_1[n-3] \\ &= 2x_1[n-2] + 4x_1[n-3] + \frac{1}{2}(2x_1[n-3] + 4x_1[n-4]) \\ &= 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4] \end{aligned}$$

The input-output relationship for S is

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

(b) The input-output relationship does not change if the order in which S_1 and S_2 are connected in series is reversed. We can easily prove this by assuming that S_1 follows S_2. In this case, the signal $x_1[n]$, which is the input to S_1, is the same as $y_2[n]$. Therefore,

$$\begin{aligned} y_1[n] &= 2x_1[n] + 4x_1[n-1] \\ &= 2y_2[n] + 4y_2[n-1] \\ &= 2(x_2[n-2] + \frac{1}{2}x_2[n-3]) + 4(x_2[n-3] + \frac{1}{2}x_2[n-4]) \\ &= 2x_2[n-2] + 5x_2[n-3] + 2x_2[n-4] \end{aligned}$$

The input-output relationship for S is once again

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

1.16. (a) The system is not memoryless because $y[n]$ depends on past values of $x[n]$.

(b) The output of the system will be $y[n] = \delta[n]\delta[n-2] = 0$.

(c) From the result of part (b), we may conclude that the system output is always zero for inputs of the form $\delta[n-k]$, $k \in \mathcal{I}$. Therefore, the system is not invertible.

1.17. (a) The system is not causal because the output $y(t)$ at some time may depend on future values of $x(t)$. For instance, $y(-\pi) = x(0)$.

(b) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \longrightarrow y_1(t) = x_1(\sin(t))$$

$$x_2(t) \longrightarrow y_2(t) = x_2(\sin(t))$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is,

$$x_3(t) = ax_1(t) + bx_2(t)$$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t)$ is

$$\begin{aligned} y_3(t) &= x_3(\sin(t)) \\ &= ax_1(\sin(t)) + bx_2(\sin(t)) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

Therefore, the system is linear.

1.18. (a) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.

$$x_1[n] \longrightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

$$x_2[n] \longrightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is,

$$x_3[n] = ax_1[n] + bx_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output $y_3[n]$ is

$$\begin{aligned} y_3[n] &= \sum_{k=n-n_0}^{n+n_0} x_3[k] \\ &= \sum_{k=n-n_0}^{n+n_0} (ax_1[k] + bx_2[k]) = a\sum_{k=n-n_0}^{n+n_0} x_1[k] + b\sum_{k=n-n_0}^{n+n_0} x_2[k] \\ &= ay_1[n] + by_2[n] \end{aligned}$$

Therefore, the system is linear.

(b) Consider an arbitrary input $x_1[n]$. Let

$$y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

be the corresponding output. Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n-n_1]$$

The output corresponding to this input is

$$y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k] = \sum_{k=n-n_0}^{n+n_0} x_1[k-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k]$$

Also note that

$$y_1[n-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k].$$

Therefore,

$$y_2[n] = y_1[n-n_1]$$

This implies that the system is time-invariant.

(c) If $|x[n]| < B$, then

$$y[n] \le (2n_0+1)B$$

Therefore, $C \le (2n_0+1)B$.

1.19. (a) (i) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \longrightarrow y_1(t) = t^2x_1(t-1)$$

$$x_2(t) \longrightarrow y_2(t) = t^2x_2(t-1)$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is,

$$x_3(t) = ax_1(t) + bx_2(t)$$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t)$ is

$$\begin{aligned} y_3(t) &= t^2x_3(t-1) \\ &= t^2(ax_1(t-1) + bx_2(t-1)) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

Therefore, the system is **linear**.

(ii) Consider an arbitrary input $x_1(t)$. Let

$$y_1(t) = t^2x_1(t-1)$$

be the corresponding output. Consider a second input $x_2(t)$ obtained by shifting $x_1(t)$ in time:

$$x_2(t) = x_1(t-t_0)$$

The output corresponding to this input is

$$y_2(t) = t^2x_2(t-1) = t^2x_1(t-1-t_0)$$

Also note that

$$y_1(t-t_0) = (t-t_0)^2x_1(t-1-t_0) \neq y_2(t)$$

Therefore, the system is **not time-invariant**.

(b) (i) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.

$$x_1[n] \longrightarrow y_1[n] = x_1^2[n-2]$$

$$x_2[n] \longrightarrow y_2[n] = x_2^2[n-2]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is,

$$x_3[n] = ax_1[n] + bx_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output $y_3[n]$ is

$$\begin{aligned} y_3[n] &= x_3^2[n-2] \\ &= (ax_1[n-2] + bx_2[n-2])^2 \\ &= a^2x_1^2[n-2] + b^2x_2^2[n-2] + 2abx_1[n-2]x_2[n-2] \\ &\neq ay_1[n] + by_2[n] \end{aligned}$$

Therefore, the system is **not linear**.

(ii) Consider an arbitrary input $x_1[n]$. Let

$$y_1[n] = x_1^2[n-2]$$

be the corresponding output. Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n-n_0]$$

The output corresponding to this input is

$$y_2[n] = x_2^2[n-2] = x_1^2[n-2-n_0]$$

Also note that

$$y_1[n-n_0] = x_1^2[n-2-n_0]$$

Therefore,

$$y_2[n] = y_1[n-n_0]$$

This implies that the system is **time-invariant**.

(c) (i) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.

$$x_1[n] \longrightarrow y_1[n] = x_1[n+1] - x_1[n-1]$$

$$x_2[n] \longrightarrow y_2[n] = x_2[n+1] - x_2[n-1]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is,

$$x_3[n] = ax_1[n] + bx_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output $y_3[n]$ is

$$\begin{aligned} y_3[n] &= x_3[n+1] - x_3[n-1] \\ &= ax_1[n+1] + bx_1[n+1] - ax_1[n-1] - bx_2[n-1] \\ &= a(x_1[n+1] - x_1[n-1]) + b(x_2[n+1] - x_2[n-1]) \\ &= ay_1[n] + by_2[n] \end{aligned}$$

Therefore, the system is **linear**.

(ii) Consider an arbitrary input $x_1[n]$. Let

$$y_1[n] = x_1[n+1] - x_1[n-1]$$

be the corresponding output. Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n-n_0]$$

The output corresponding to this input is

$$y_2[n] = x_2[n+1] - x_2[n-1] = x_1[n+1-n_0] - x_1[n-1-n_0]$$

Also note that

$$y_1[n-n_0] = x_1[n+1-n_0] - x_1[n-1-n_0]$$

Therefore,

$$y_2[n] = y_1[n-n_0]$$

This implies that the system is **time-invariant**.

(d) (i) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \longrightarrow y_1(t) = Od\{x_1(t)\}$$
$$x_2(t) \longrightarrow y_2(t) = Od\{x_2(t)\}$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is,

$$x_3(t) = ax_1(t) + bx_2(t)$$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t)$ is

$$\begin{aligned} y_3(t) &= Od\{x_3(t)\} \\ &= Od\{ax_1(t) + bx_2(t)\} \\ &= aOd\{x_1(t)\} + bOd\{x_2(t)\} = ay_1(t) + by_2(t) \end{aligned}$$

Therefore, the system is **linear**.

(ii) Consider an arbitrary input $x_1(t)$. Let

$$y_1(t) = Od\{x_1(t)\} = \frac{x_1(t) - x_1(-t)}{2}$$

be the corresponding output. Consider a second input $x_2(t)$ obtained by shifting $x_1[n]$ in time:

$$x_2(t) = x_1(t - t_0)$$

The output corresponding to this input is

$$\begin{aligned} y_2(t) &= Od\{x_2(t)\} = \frac{x_2(t) - x_2(-t)}{2} \\ &= \frac{x_1(t-t_0) - x_1(-t-t_0)}{2} \end{aligned}$$

Also note that

$$y_1(t-t_0) = \frac{x_1(t-t_0) - x_1(-t+t_0)}{2} \neq y_2(t)$$

Therefore, the system is **not time-invariant**.

1.20. (a) Given

$$x(t) = e^{j2t} \longrightarrow y(t) = e^{j3t}$$
$$x(t) = e^{-j2t} \longrightarrow y(t) = e^{-j3t}$$

Since the system is linear,

$$x_1(t) = \frac{1}{2}(e^{j2t} + e^{-j2t}) \longrightarrow y_1(t) = \frac{1}{2}(e^{j3t} + e^{-j3t})$$

Therefore,

$$x_1(t) = \cos(2t) \longrightarrow y_1(t) = \cos(3t)$$

(b) We know that

$$x_2(t) = \cos\left(2(t - \frac{1}{2})\right) = \frac{e^{-j}e^{j2t} + e^{j}e^{-j2t}}{2}$$

Using the linearity property, we may once again write

$$x_1(t) = \frac{1}{2}(e^{-j}e^{j2t} + eje^{-j2t}) \longrightarrow y_1(t) = \frac{1}{2}(e^{-j}e^{j3t} + e^{j}e^{-j3t}) = \cos(3t-1)$$

Therefore,

$$x_1(t) = \cos(2(t-1/2)) \longrightarrow y_1(t) = \cos(3t-1)$$

1.21. The signals are sketched in Figure S1.21.

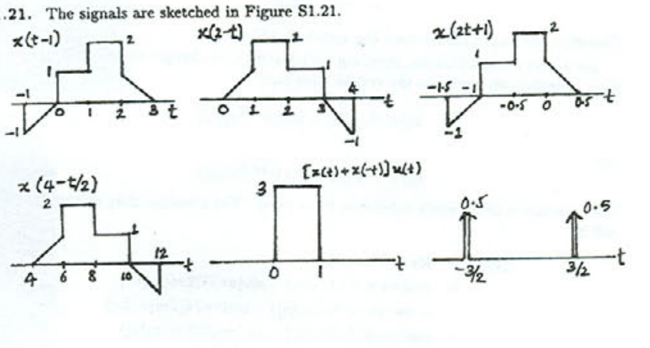

Figure S1.21

1.22. The signals are sketched in Figure S1.22.

1.23. The even and odd parts are sketched in Figure S1.23.

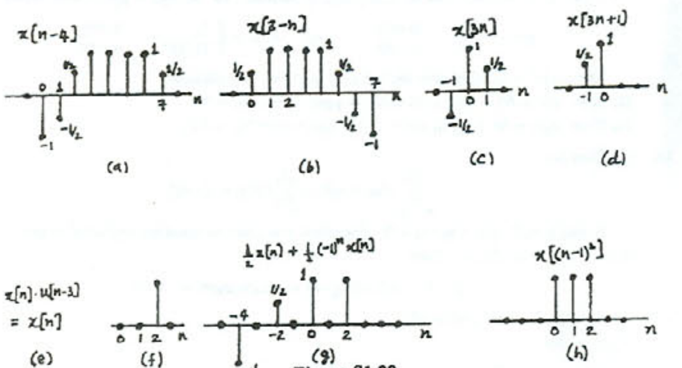

Figure S1.22

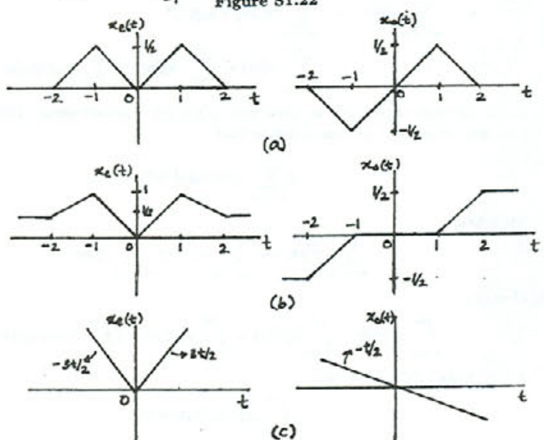

Figure S1.23

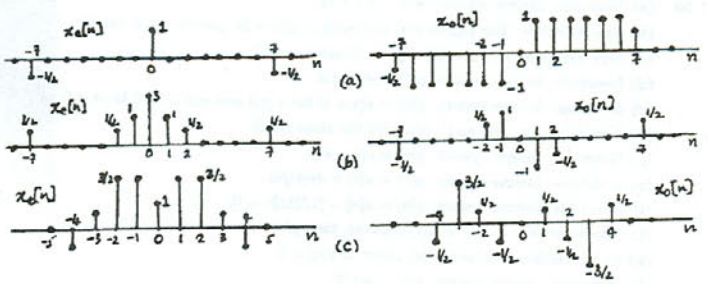

Figure S1.24

1.24. The even and odd parts are sketched in Figure S1.24.

1.25. (a) Periodic, period $= 2\pi/(4) = \pi/2$.
(b) Periodic, period $= 2\pi/(\pi) = 2$.
(c) $x(t) = [1 + \cos(4t - 2\pi/3)]/2$. Periodic, period $= 2\pi/(4) = \pi/2$.
(d) $x(t) = \cos(4\pi t)/2$. Periodic, period $= 2\pi/(4\pi) = 1/2$.
(e) $x(t) = [\sin(4\pi t)u(t) - \sin(4\pi t)u(-t)]/2$. Not periodic.
(f) Not periodic.

1.26. (a) Periodic, period $= 7$.
(b) Not periodic.
(c) Periodic, period $= 8$.
(d) $x[n] = (1/2)[\cos(3\pi n/4) + \cos(\pi n/4)]$. Periodic, period $= 8$.
(e) Periodic, period $= 16$.

1.27. (a) Linear, stable.
(b) Memoryless, linear, causal, stable.
(c) Linear
(d) Linear, causal, stable.
(e) Time invariant, linear, causal, stable.
(f) Linear, stable.
(g) Time invariant, linear, causal.

1.28. (a) Linear, stable.

(b) Time invariant, linear, causal, stable.

(c) Memoryless, linear, causal.

(d) Linear, stable.

(e) Linear, stable.

(f) Memoryless, linear, causal, stable.

(g) Linear, stable.

1.29. (a) Consider two inputs to the system such that

$$x_1[n] \xrightarrow{S} y_1[n] = \mathcal{R}e\{x_1[n]\} \quad \text{and} \quad x_2[n] \xrightarrow{S} y_2[n] = \mathcal{R}e\{x_2[n]\}.$$

Now consider a third input $x_3[n] = x_1[n] + x_2[n]$. The corresponding system output will be

$$\begin{aligned} y_3[n] &= \mathcal{R}e\{x_3[n]\} \\ &= \mathcal{R}e\{x_1[n] + x_2[n]\} \\ &= \mathcal{R}e\{x_1[n]\} + \mathcal{R}e\{x_2[n]\} \\ &= y_1[n] + y_2[n] \end{aligned}$$

Therefore, we may conclude that the system is additive.

Let us now assume that the input-output relationship is changed to $y[n] = \mathcal{R}e\{e^{j\pi/4}x[n]\}$. Also, consider two inputs to the system such that

$$x_1[n] \xrightarrow{S} y_1[n] = \mathcal{R}e\{e^{j\pi/4}x_1[n]\}$$

and

$$x_2[n] \xrightarrow{S} y_2[n] = \mathcal{R}e\{e^{j\pi/4}x_2[n]\}.$$

Now consider a third input $x_3[n] = x_1[n] + x_2[n]$. The corresponding system output will be

$$\begin{aligned} y_3[n] &= \mathcal{R}e\{e^{j\pi/4}x_3[n]\} \\ &= \cos(\pi n/4)\mathcal{R}e\{x_3[n]\} - \sin(\pi n/4)\mathcal{I}m\{x_3[n]\} \\ &\quad + \cos(\pi n/4)\mathcal{R}e\{x_1[n]\} - \sin(\pi n/4)\mathcal{I}m\{x_1[n]\} \\ &+ \cos(\pi n/4)\mathcal{R}e\{x_2[n]\} - \sin(\pi n/4)\mathcal{I}m\{x_2[n]\} \\ &= \mathcal{R}e\{e^{j\pi/4}x_1[n]\} + \mathcal{R}e\{e^{j\pi/4}x_2[n]\} \\ &= y_1[n] + y_2[n] \end{aligned}$$

Therefore, we may conclude that the system is additive.

(b) (i) Consider two inputs to the system such that

$$x_1(t) \xrightarrow{S} y_1(t) = \frac{1}{x_1(t)}\left[\frac{dx_1(t)}{dt}\right]^2 \quad \text{and} \quad x_2(t) \xrightarrow{S} y_2(t) = \frac{1}{x_1(t)}\left[\frac{dx_2(t)}{dt}\right]^2.$$

Now consider a third input $x_3(t) = x_1(t) + x_2(t)$. The corresponding system output will be

$$\begin{aligned} y_3(t) &= \frac{1}{x_3(t)}\left[\frac{dx_3(t)}{dt}\right]^2 \\ &= \frac{1}{x_1(t) + x_2(t)}\left[\frac{d[x_1(t) + x_2(t)]}{dt}\right]^2 \\ &\neq y_1(t) + y_2(t) \end{aligned}$$

Therefore, we may conclude that the system is not additive.

Now consider a fourth input $x_4(t) = ax_1(t)$. The corresponding output will be

$$\begin{aligned} y_4(t) &= \frac{1}{x_4(t)}\left[\frac{dx_4(t)}{dt}\right]^2 \\ &= \frac{1}{ax_1(t)}\left[\frac{d[ax_1(t)]}{dt}\right]^2 \\ &= \frac{a}{x_1(t)}\left[\frac{dx_1(t)}{dt}\right]^2 \\ &= ay_1(t) \end{aligned}$$

Therefore, the system is homogeneous.

(ii) This system is not additive. Consider the following example. Let $x_1[n] = 2\delta[n+2] + 2\delta[n+1] + 2\delta[n]$ and $x_2[n] = \delta[n+2] + 2\delta[n+1] + 3\delta[n]$. The corresponding outputs evaluated at $n = 0$ are

$$y_1[0] = 2 \quad \text{and} \quad y_2[0] = 3/2.$$

Now consider a third input $x_3[n] = x_1[n] + x_2[n] = 3\delta[n+2] + 4\delta[n+1] + 5\delta[n]$. The corresponding output evaluated at $n = 0$ is $y_3[0] = 15/4$. Clearly, $y_3[0] \neq y_1[0] + y_2[0]$. This implies that the system in not additive.

No consider an input $x_4[n]$ which leads to the output $y_4[n]$. We know that

$$y_4[n] = \begin{cases} \frac{x_4[n]x_4[n-2]}{x_4[n-1]}, & x_4[n-1] \neq 0 \\ 0, & \text{otherwise} \end{cases}.$$

Let us now consider another input $x_5[n] = ax_4[n]$. The corresponding output is

$$y_5[n] = \begin{cases} a\frac{x_4[n]x_4[n-2]}{x_4[n-1]}, & x_4[n-1] \neq 0 \\ 0, & \text{otherwise} \end{cases} = ay_4[n].$$

Therefore, the system is homogeneous.

1.30. (a) Invertible. Inverse system: $y(t) = x(t+4)$.

(b) Non invertible. The signals $x(t)$ and $x_1(t) = x(t) + 2\pi$ give the same output.

(c) Non invertible. $\delta[n]$ and $2\delta[n]$ give the same output.

(d) Invertible. Inverse system: $y(t) = dx(t)/dt$.

(e) Invertible. Inverse system: $y[n] = x[n+1]$ for $n \geq 0$ and $y[n] = x[n]$ for $n < 0$.

(f) Non invertible. $x[n]$ and $-x[n]$ give the same result.

(g) Invertible. Inverse system: $y[n] = x[1-n]$.

(h) Invertible. Inverse system: $y(t) = x(t) + dx(t)/dt$.

(i) Invertible. Inverse system: $y[n] = x[n] - (1/2)x[n-1]$.

(j) Non invertible. If $x(t)$ is any constant, then $y(t) = 0$.

(k) Non invertible. $\delta[n]$ and $2\delta[n]$ result in $y[n] = 0$.

(l) Invertible. Inverse system: $y(t) = x(t/2)$.

(m) Non invertible. $x_1[n] = \delta[n] + \delta[n-1]$ and $x_2[n] = \delta[n]$ give $y[n] = \delta[n]$.

(n) Invertible. Inverse system: $y[n] = x[2n]$.

1.31. (a) Note that $x_2(t) = x_1(t) - x_1(t-2)$. Therefore, using linearity we get $y_2(t) = y_1(t) - y_1(t-2)$. This is as shown in Figure S1.31.

(b) Note that $x_3(t) = x_1(t) + x_1(t+1)$. Therefore, using linearity we get $y_3(t) = y_1(t) + y_1(t+1)$. This is as shown in Figure S1.31.

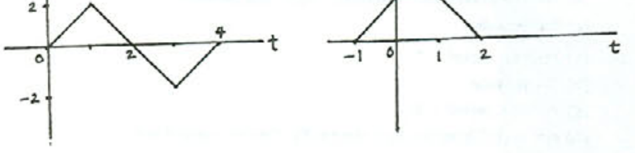

Figure S1.31

1.32. All statements are true.

(1) $x(t)$ periodic with period T; $y_1(t)$ periodic, period $T/2$.

(2) $y_1(t)$ periodic, period T; $x(t)$ periodic, period $2T$.

(3) $x(t)$ periodic, period T; $y_2(t)$ periodic, period $2T$.

(4) $y_2(t)$ periodic, period T; $x(t)$ periodic, period $T/2$.

1.33. (1) True. $x[n] = x[n+N]$; $y_1[n] = y_1[n+N_0]$. i.e. periodic with $N_0 = N/2$ if N is even, and with period $N_0 = N$ if N is odd.

(2) False. $y_1[n]$ periodic does no imply $x[n]$ is periodic. i.e. let $x[n] = g[n] + h[n]$ where

$$g[n] = \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \quad \text{and} \quad h[n] = \begin{cases} 0, & n \text{ even} \\ (1/2)^n, & n \text{ odd} \end{cases}.$$

Then $y_1[n] = x[2n]$ is periodic but $x[n]$ is clearly not periodic.

(3) True. $x[n+N] = x[n]$; $y_2[n+N_0] = y_2[n]$ where $N_0 = 2N$

(4) True. $y_2[n+N] = y_2[n]$; $x[n+N_0] = x[n]$ where $N_0 = N/2$

1.34. (a) Consider

$$\sum_{n=-\infty}^{\infty} x[n] = x[0] + \sum_{n=1}^{\infty}\{x[n] + x[-n]\}.$$

If $x[n]$ is odd, $x[n] + x[-n] = 0$. Therefore, the given summation evaluates to zero.

(b) Let $y[n] = x_1[n]x_2[n]$. Then

$$y[-n] = x_1[-n]x_2[-n] = -x_1[n]x_2[n] = -y[n].$$

This implies that $y[n]$ is odd.

(c) Consider

$$\begin{aligned} \sum_{n=-\infty}^{\infty} x^2[n] &= \sum_{n=-\infty}^{\infty}\{x_e[n] + x_o[n]\}^2 \\ &= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n] + 2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n]. \end{aligned}$$

Using the result of part (b), we know that $x_e[n]x_o[n]$ is an odd signal. Therefore, using the result of part (a) we may conclude that

$$2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 0.$$

Therefore,

$$\sum_{n=-\infty}^{\infty} x^2[n] == \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n].$$

(d) Consider

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt + 2\int_{-\infty}^{\infty} x_e(t)x_o(t)dt.$$

Again, since $x_e(t)x_o(t)$ is odd,

$$\int_{-\infty}^{\infty} x_e^2(t)x_o(t)dt = 0.$$

Therefore,

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt.$$

1.35. We want to find the smallest N_0 such that $m(2\pi/N)N_0 = 2\pi k$ or $N_0 = kN/m$, where k is an integer. If N_0 has to be an integer, then N must be a multiple of m/k and m/k must be an integer. This implies that m/k is a divisor of both m and N. Also, if we want the smallest possible N_0, then m/k should be the GCD of m and N. Therefore, $N_0 = N/\gcd(m, N)$.

1.36. (a) If $x[n]$ is periodic $e^{j\omega_0(n+N)T} = e^{j\omega_0 nT}$, where $\omega_0 = 2\pi/T_0$. This implies that

$$\frac{2\pi}{T_0}NT = 2\pi k \quad \Rightarrow \quad \frac{T}{T_0} = \frac{k}{N} = \text{a rational number.}$$

(b) If $T/T_0 = p/q$ then $x[n] = e^{j2\pi n(p/q)}$. The fundamental period is $q/\gcd(p,q)$ and the fundamental frequency is

$$\frac{2\pi}{q}\gcd(p,q) = \frac{2\pi}{p}\frac{p}{q}\gcd(p,q) = \frac{\omega_0}{p}\gcd(p,q) = \frac{\omega_0 T}{p}\gcd(p,q).$$

(c) $p/\gcd(p,q)$ periods of $x(t)$ are needed.

1.37. (a) From the definition of $\phi_{xy}(t)$, we have

$$\begin{aligned}\phi_{xy}(t) &= \int_{-\infty}^{\infty} x(t+\tau)y(\tau)d\tau \\ &= \int_{-\infty}^{\infty} y(-t+\tau)x(\tau)d\tau \\ &= \phi_{yx}(-t).\end{aligned}$$

(b) Note from part (a) that $\phi_{xx}(t) = \phi_{xx}(-t)$. This implies that $\phi_{xx}(t)$ is even. Therefore, the odd part of $\phi_{xx}(t)$ is zero.

(c) Here, $\phi_{xy}(t) = \phi_{xx}(t-T)$ and $\phi_{yy}(t) = \phi_{xx}(t)$.

1.38. (a) We know that $2\delta_\Delta(2t) = \delta_{\Delta/2}(t)$. Therefore,

$$\lim_{\Delta\to 0}\delta_\Delta(2t) = \lim_{\Delta\to 0}\frac{1}{2}\delta_{\Delta/2}(t).$$

This implies that

$$\delta(2t) = \frac{1}{2}\delta(t).$$

(b) The plots are as shown in Figure S1.38.

1.39. We have

$$\lim_{\Delta\to 0} u_\Delta(t)\delta(t) = \lim_{\Delta\to 0} u_\Delta(0)\delta(t) = 0.$$

Also,

$$\lim_{\Delta\to 0} u_\Delta(t)\delta_\Delta(t) = \frac{1}{2}\delta(t).$$

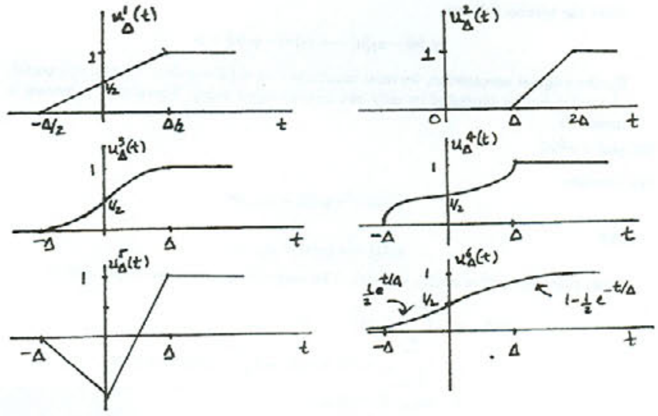

Figure S1.38

We have

$$g(t) = \int_{-\infty}^{\infty} u(\tau)\delta(t-\tau)d\tau = \int_{0}^{\infty} u(\tau)\delta(t-\tau)d\tau.$$

Therefore,

$$g(t) = \begin{cases} 0, & t < 0 \quad \because \delta(t-\tau) = 0 \\ 1, & t > 0 \quad \because u(\tau)\delta(t-\tau) = \delta(t-\tau) \\ \text{undefined} & \text{for } t = 0 \end{cases}$$

1.40. (a) If a system is additive, then

$$0 = x(t) - x(t) \longrightarrow y(t) - y(t) = 0.$$

Also, if a system is homogeneous, then

$$0 = 0.x(t) \longrightarrow y(t).0 = 0.$$

(b) $y(t) = x^2(t)$ is such a system.

(c) No. For example, consider $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$ with $x(t) = u(t) - u(t-1)$. Then $x(t) = 0$ for $t > 1$, but $y(t) = 1$ for $t > 1$.

1.41. (a) $y[n] = 2x[n]$. Therefore, the system is time invariant.

(b) $y[n] = (2n-1)x[n]$. This is not time-invariant because $y[n-N_0] \neq (2n-1)x[n-N_0]$.

(c) $y[n] = x[n]\{1 + (-1)^n + 1 + (-1)^{n-1}\} = 2x[n]$. Therefore, the system is time invariant.

1.42. (a) Consider two systems S_1 and S_2 connected in series. Assume that if $x_1(t)$ and $x_2(t)$ are the inputs to S_1, then $y_1(t)$ and $y_2(t)$ are the outputs, respectively. Also, assume that if $y_1(t)$ and $y_2(t)$ are the inputs to S_2, then $z_1(t)$ and $z_2(t)$ are the outputs, respectively. Since S_1 is linear, we may write

$$ax_1(t) + bx_2(t) \xrightarrow{S_1} ay_1(t) + by_2(t),$$

where a and b are constants. Since S_2 is also linear, we may write

$$ay_1(t) + by_2(t) \xrightarrow{S_2} az_1(t) + bz_2(t),$$

We may therefore conclude that

$$ax_1(t) + bx_2(t) \xrightarrow{S_1,S_2} az_1(t) + bz_2(t).$$

Therefore, the series combination of S_1 and S_2 is linear.

Since S_1 is time invariant, we may write

$$x_1(t - T_0) \xrightarrow{S_1} y_1(t - T_0)$$

and

$$y_1(t - T_0) \xrightarrow{S_2} z_1(t - T_0).$$

Therefore,

$$x_1(t - T_0) \xrightarrow{S_1,S_2} z_1(t - T_0).$$

Therefore, the series combination of S_1 and S_2 is time invariant.

(b) False. Let $y(t) = x(t) + 1$ and $z(t) = y(t) - 1$. These correspond to two nonlinear systems. If these systems are connected in series, then $z(t) = x(t)$ which is a linear system.

(c) Let us name the output of system 1 as $w[n]$ and the output of system 2 as $z[n]$. Then,

$$\begin{aligned} y[n] &= z[2n] = w[2n] + \frac{1}{2}w[2n-1] + \frac{1}{4}w[2n-2] \\ &= x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2] \end{aligned}$$

The overall system is linear and time-invariant.

1.43. (a) We have

$$x(t) \xrightarrow{S} y(t).$$

Since S is time-invariant,

$$x(t - T) \xrightarrow{S} y(t - T).$$

Now, if $x(t)$ is periodic with period T, $x(t) = x(t-T)$. Therefore, we may conclude that $y(t) = y(t-T)$. This implies that $y(t)$ is also periodic with period T. A similar argument may be made in discrete time.

(b)

1.44. (a) Assumption: If $x(t) = 0$ for $t < t_0$, then $y(t) = 0$ for $t < t_0$. To prove that: The system is causal.

Let us consider an arbitrary signal $x_1(t)$. Let us consider another signal $x_2(t)$ which is the same as $x_1(t)$ for $t < t_0$. But for $t > t_0$, $x_2(t) \neq x_1(t)$. Since the system is linear,

$$x_1(t) - x_2(t) \longrightarrow y_1(t) - y_2(t).$$

Since $x_1(t) - x_2(t) = 0$ for $t < t_0$, by our assumption $y_1(t) - y_2(t) = 0$ for $t < t_0$. This implies that $y_1(t) = y_2(t)$ for $t < t_0$. In other words, the output is not affected by input values for $t \geq t_0$. Therefore, the system is causal.

Assumption: The system is causal. To prove that: If $x(t) = 0$ for $t < t_0$, then $y(t) = 0$ for $t < t_0$.

Let us assume that the signal $x(t) = 0$ for $t < t_0$. Then we may express $x(t)$ as $x(t) = x_1(t) - x_2(t)$, where $x_1(t) = x_2(t)$ for $t < t_0$. Since the system is linear, the output to $x(t)$ will be $y(t) = y_1(t) - y_2(t)$. Now, since the system is causal, $y_1(t) = y_2(t)$ for $t < t_0$ implies that $y_1(t) = y_2(t)$ for $t < t_0$. Therefore, $y(t) = 0$ for $t < t_0$.

(b) Consider $y(t) = x(t)x(t+1)$. Now, $x(t) = 0$ for $t < t_0$ implies that $y(t) = 0$ for $t < t_0$. Note that the system is nonlinear and non-causal.

(c) Consider $y(t) = x(t) + 1$. This system is nonlinear and causal. This does not satisfy the condition of part (a).

(d) Assumption: The system is invertible. To prove that: $y[n] = 0$ for all n only if $x[n] = 0$ for all n.

Consider

$$x[n] = 0 \longrightarrow y[n].$$

Since the system is linear,

$$2x[n] = 0 \longrightarrow 2y[n].$$

Since the input has not changed in the two above equations, we require that $y[n] = 2y[n]$. This implies that $y[n] = 0$. Since we have assumed that the system is invertible, only one input could have led to this particular output. That input must be $x[n] = 0$.

Assumption: $y[n] = 0$ for all n if $x[n] = 0$ for all n. To prove that: The system is invertible.

Suppose that

$$x_1[n] \longrightarrow y_1[n]$$

and

$$x_2[n] \longrightarrow y_1[n].$$

Since the system is linear,

$$x_1[n] - x_2[n] \longrightarrow y_1[n] - y_1[n] = 0.$$

By the original assumption, we must conclude that $x_1[n] = x_2[n]$. That is, any particular $y_1[n]$ can be produced by only one distinct input $x_1[n]$. Therefore, the system is invertible.

(e) $y[n] = x^2[n]$.

1.45. (a) Consider

$$x_1(t) \xrightarrow{S} y_1(t) = \phi_{hx_1}(t)$$

and

$$x_2(t) \xrightarrow{S} y_2(t) = \phi_{hx_2}(t).$$

Now, consider $x_3(t) = ax_1(t) + bx_2(t)$. The corresponding system output will be

$$\begin{aligned} y_3(t) &= \int_{-\infty}^{\infty} x_3(\tau)h(t+\tau)d\tau \\ &= a\int_{-\infty}^{\infty} x_1(\tau)h(t+\tau)d\tau + b\int_{-\infty}^{\infty} x_2(\tau)h(t+\tau)d\tau \\ &= a\phi_{hx_1}(t) + b\phi_{hx_2}(t) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

Therefore, S is linear.

Now, consider $x_4(t) = x_1(t-T)$. The corresponding system output will be

$$\begin{aligned} y_4(t) &= \int_{-\infty}^{\infty} x_4(\tau)h(t+\tau)d\tau \\ &= \int_{-\infty}^{\infty} x_1(\tau - T)h(t+\tau)d\tau \\ &= \int_{-\infty}^{\infty} x_1(\tau)h(t+\tau+T)d\tau \\ &= \phi_{hx_1}(t+T) \end{aligned}$$

Clearly, $y_4(t) \neq y_1(t-T)$. Therefore, the system is not time-invariant.

The system is definitely not causal because the output at any time depends on future values of the input signal $x(t)$.

(b) The system will then be linear, time invariant and non-causal.

1.46. The plots are as in Figure S1.46.

1.47. (a) The overall response of the system of Figure P1.47(a) = (the response of the system to $x[n] + x_1[n]$) − the response of the system to $x_1[n]$ = (Response of a linear system L to $x[n] + x_1[n]$+ zero input response of S) − (Response of a linear system L to $x_1[n]$+ zero input response of S) = (Response of a linear system L to $x[n]$).

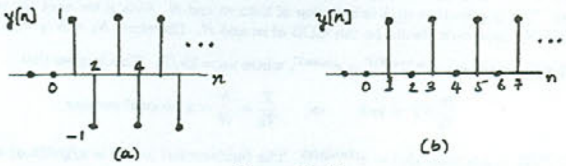

Figure S1.46

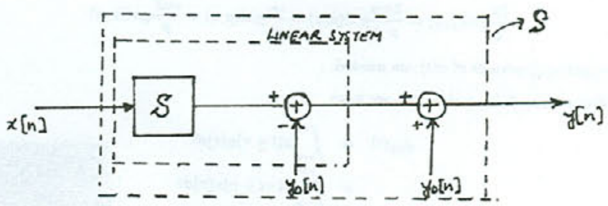

Figure S1.47

(b) If $x_1[n] = 0$ for all n, then $y_1[n]$ will be the zero-input response $y_0[n]$. S may then be redrawn as shown in Figure S1.47. This is the same as Figure 1.48.

(c) (i) Incrementally linear.

$$x[n] \longrightarrow x[n] + 2x[n+1] \quad \text{and} \quad y_0[n] = n$$

(ii) Incrementally linear.

$$x[n] \longrightarrow \begin{cases} 0, & n \text{ even} \\ \sum_{k=-\infty}^{(n-1)/2} x[k], & n \text{ odd.} \end{cases}$$

and

$$y_0[n] = \begin{cases} n/2, & n \text{ even} \\ (n-1)/2, & n \text{ odd.} \end{cases}$$

(iii) Not incrementally linear. Eg. choose $y_0[n] = 3$. Then

$$y[n] - y_0[n] = \begin{cases} x[n] - x[n-1], & x[0] \geq 0 \\ x[n] - x[n-1] - 6, & x[0] < 0. \end{cases}$$

Still non-linear: eg.: If $x_1[n] = -\delta[n]$ and $x_2[n] = -2\delta[n]$, then $y_1[n] = -\delta[n] + \delta[n-1] - 6$ and $y_2[n] = -2\delta[n] + 2\delta[n-1] - 6 \neq 2y_1[n]$.

(iv) Incrementally linear.

$$x(t) \longrightarrow x(t) + t\,dx(t)/dt - 1 \quad \text{and} \quad y_0(t) = 1.$$

(v) Incrementally linear

$$x[n] \longrightarrow 2\cos(\pi n)x[n] \quad \text{and} \quad y_0[n] = \cos^2(\pi n)$$

(d) Let $x[n] \xrightarrow{S} y[n]$ and $x[n] \xrightarrow{L} z[n]$. Then, $y[n] = z[n] + c$. For time invariance, we require that when the input is $x[n-n_0]$, the output be

$$y[n-n_0] = z[n-n_0] + c.$$

This implies that we require

$$x[n-n_0] \xrightarrow{L} z[n-n_0]$$

which in turn implies that L should be time invariant. We also require that $y_0[n] = c$ =constant independent of n.

1.48. We have

$$z_0 = r_0 e^{j\theta_0} = r_0\cos\theta_0 + jr_0\sin\theta_0 = x_0 + jy_0$$

(a) $z_1 = x_0 - jy_0$

(b) $z_2 = \sqrt{x_0^2 + y_0^2}$

(c) $z_3 = -x_0 - jy_0 = -z_0$

(d) $z_4 = -x_0 + jy_0$

(e) $z_5 = x_0 + jy_0$

The plots for the points are as shown in the Figure S1.48.

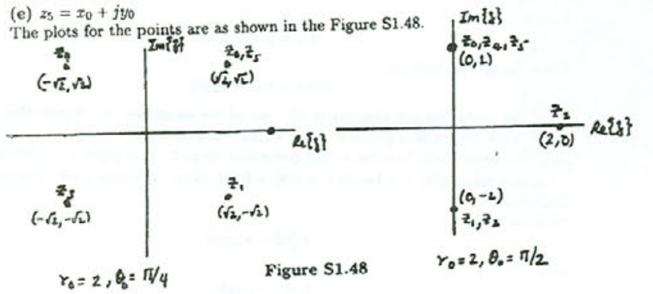

Figure S1.48

1.49. (a) Here, $r = \sqrt{1+3} = 2$. Also, $\cos\theta = 1/2$, $\sin\theta = \sqrt{3}/2$. This implies that $\theta = \pi/3$. Therefore, $1 + j\sqrt{3} = 2e^{j\pi/3}$.

(b) $5e^{j\pi}$

(c) $5\sqrt{2}e^{j5\pi/4}$

(d) $5e^{j\tan^{-1}(4/3)} = 5e^{j(53.13^\circ)}$

(e) $8e^{-j\pi}$

(f) $4\sqrt{2}e^{j5\pi/4}$

(g) $2\sqrt{2}e^{-j5\pi/12}$

(h) $e^{-j2\pi/3}$

(i) $e^{j\pi/6}$

(j) $\sqrt{2}e^{j11\pi/12}$

(k) $4\sqrt{2}e^{-j\pi/12}$

(l) $\frac{1}{2}e^{j\pi/3}$

Plot depicting these points is as shown in Figure S1.49.

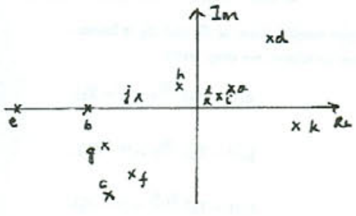

Figure S1.49

1.50. (a) $x = r\cos\theta, y = r\sin\theta$

(b) We have

$$r = \sqrt{x^2 + y^2}$$

and

$$\theta = \sin^{-1}\left[\frac{y}{\sqrt{x^2+y^2}}\right] = \cos^{-1}\left[\frac{x}{\sqrt{x^2+y^2}}\right] = \tan^{-1}\left[\frac{y}{x}\right].$$

θ is undefined if $r = 0$ and also irrelevant. θ is not unique since θ and $\theta + 2m\pi$ ($m \in$ integer) give the same results.

(c) θ and $\theta + \pi$ have the same value of tangent. We only know that the complex number is either $z_1 re^{j\theta}$ or $z_2 = re^{j(\theta+\pi)} = -z_1$.

1.51. (a) We have

$$e^{j\theta} = \cos\theta + j\sin\theta. \qquad \text{(S1.51–1)}$$

and

$$e^{-j\theta} = \cos\theta - j\sin\theta. \qquad \text{(S1.51–2)}$$

Summing eqs. (S1.51-1) and (S1.51-2) we get

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}).$$

(b) Subtracting eq. (S1.51-2) from (S1.51-1) we get

$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta}).$$

(c) We now have $e^{j(\theta+\phi)} = e^{j\theta}e^{j\phi}$. Therefore,

$$\begin{aligned}\cos(\theta+\phi) + j\sin(\theta+\phi) &= (\cos\theta\cos\phi - \sin\theta\sin\phi) \\ &+ j(\sin\theta\cos\phi + \cos\theta\sin\phi)\end{aligned} \qquad \text{(S1.51–3)}$$

Putting $\theta = \phi$ in eq. (S1.51-3), we get

$$\cos 2\theta = \cos^2\theta - \sin^2\theta.$$

Putting $\theta = -\phi$ in eq. (S1.51-3), we get

$$1 = \cos^2\theta + \sin^2\theta.$$

Adding the two above equations and simplifying

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta).$$

(d) Equating the real parts in eq. (S1.51-3) with arguments $(\theta+\phi)$ and $(\theta-\phi)$ we get

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

and

$$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi.$$

Subtracting the two above equations, we obtain

$$\sin\theta\sin\phi = \frac{1}{2}[\cos(\theta-\phi) - \cos(\theta+\phi)].$$

(e) Equating imaginary parts in in eq. (S1.51-3), we get

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi.$$

1.52. (a) $zz^* = re^{j\theta}re^{-j\theta} = r^2$

(b) $z/z^* = re^{j\theta}r^{-1}e^{j\theta} = e^{j2\theta}$

(c) $z + z^* = x + jy + x - jy = 2x = 2\mathcal{R}e\{z\}$

(d) $z - z^* = x + jy - x + jy = 2jy = 2\mathcal{I}m\{z\}$

(e) $(z_1 + z_2)^* = ((x_1 + x_2) + j(y_1 + y_2))^* = x_1 - jy_1 + x_2 - jy_2 = z_1^* + z_2^*$

(f) Consider $(az_1z_2)^*$ for $a > 0$.

$$(az_1z_2)^* = (ar_1r_2e^{j(\theta_1+\theta_2)})^* = ar_1e^{-j\theta_1}r_2e^{-j\theta_2} = az_1^*z_2^*.$$

For $a < 0$, $a = |a|e^{j\pi}$. Therefore,

$$(az_1z_2)^* = (|a|r_1r_2e^{j(\theta_1+\theta_2+\pi)})^* = |a|e^{-j\pi}r_1e^{-j\theta_1}r_2e^{-j\theta_2} = az_1^*z_2^*.$$

(g) For $|z_2| \neq 0$,

$$\left(\frac{z_1}{z_2}\right)^* = \frac{r_1}{r_2}e^{-j\theta_1}e^{j\theta_2} = \frac{r_1e^{-j\theta_1}}{r_2e^{-j\theta_2}} = \frac{z_1^*}{z_2^*}.$$

(h) From (c), we get

$$\mathcal{R}e\{\frac{z_1}{z_2}\} = \frac{1}{2}\left[\left(\frac{z_1}{z_2}\right) + \left(\frac{z_1}{z_2}\right)^*\right].$$

Using (g) on this, we get

$$\mathcal{R}e\{\frac{z_1}{z_2}\} = \frac{1}{2}\left[\left(\frac{z_1}{z_2}\right) + \left(\frac{z_1^*}{z_2^*}\right)\right] = \frac{1}{2}\left[\frac{z_1z_2^* + z_1^*z_2}{z_2z_2^*}\right].$$

1.53. (a) $(e^z)^* = (e^xe^{jy})^* = e^xe^{-jy} = e^{x-jy} = e^{z^*}$.

(b) Let $z_3 = z_1z_2^*$ and $z_4 = z_1^*z_2$. Then,

$$\begin{aligned}z_1z_2^* + z_1^*z_2 &= z_3 + z_3^* = 2\mathcal{R}e\{z_3\} = 2\mathcal{R}e\{z_1z_2^*\} \\ &= z_4^* + z_4 = 2\mathcal{R}e\{z_4\} = 2\mathcal{R}e\{z_1^*z_2\}\end{aligned}$$

(c) $|z| = |re^{j\theta}| = r = |re^{-j\theta}| = |z^*|$

(d) $|z_1z_2| = |r_1r_2e^{j(\theta_1+\theta_2)}| = |r_1r_2| = |r_1||r_2| = |z_1||z_2$

(e) Since $z = x + jy$, $|z| = \sqrt{x^2+y^2}$. By the triangle inequality,

$$\mathcal{R}e\{z\} = x \leq \sqrt{x^2+y^2} = |z|$$

and

$$\mathcal{I}m\{z\} = y \leq \sqrt{x^2+y^2} = |z|.$$

(f) $|z_1z_2^* + z_1^*z_2| = |2\mathcal{R}e\{z_1z_2^*\}| = |2r_1r_2\cos(\theta_1-\theta_2)| \leq 2r_1r_2 = 2|z_1z_2|$.

(g) Since $r_1 > 0, r_2 > 0$ and $-1 \leq \cos(\theta_1 - \theta_2) \leq 1$,

$$\begin{aligned}(|z_1| - |z_2|)^2 &= r_1^2 + r_2^2 - 2r_1r_2 \\ &\leq r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1-\theta_2) \\ &= |z_1 + z_2|^2\end{aligned}$$

and

$$(|z_1| + |z_2|)^2 = r_1^2 + r_2^2 + 2r_1r_2 \geq |z_1 + z_2|^2.$$

1.54. (a) For $\alpha = 1$, it is fairly obvious that

$$\sum_{n=0}^{N-1}\alpha^n = N.$$

For $\alpha \neq 1$, we may write

$$(1-\alpha)\sum_{n=0}^{N-1}\alpha^n = \sum_{n=0}^{N-1}\alpha^n - \sum_{n=0}^{N-1}\alpha^{n+1} = 1 - \alpha^N.$$

Therefore,

$$\sum_{n=0}^{N-1}\alpha^n = \frac{1-\alpha^N}{1-\alpha}.$$

(b) For $|\alpha| < 1$,

$$\lim_{N\to\infty}\alpha^N = 0.$$

Therefore, from the result of the previous part,

$$\lim_{N\to\infty}\sum_{n=0}^{N-1}\alpha^n = \sum_{n=0}^{\infty}\alpha^n = \frac{1}{1-\alpha}.$$

(c) Differentiating both sides of the result of part (b) wrt α, we get

$$\begin{aligned}\frac{d}{d\alpha}\left(\sum_{n=0}^{\infty}\alpha^N\right) &= \frac{d}{d\alpha}\left(\frac{1}{1-\alpha}\right) \\ \sum_{n=0}^{\infty}n\alpha^{n-1} &= \frac{1}{(1-\alpha)^2}\end{aligned}$$

(d) We may write

$$\sum_{n=k}^{\infty}\alpha^n = \alpha^k\sum_{n=0}^{\infty}\alpha^n = \frac{\alpha^k}{1-\alpha} \text{ for } |\alpha| < 1.$$

1.55. (a) The desired sum is

$$\sum_{n=0}^{9}e^{j\pi n/2} = \frac{1-e^{j\pi 10/2}}{1-e^{j\pi/2}} = 1 + j.$$

(b) The desired sum is

$$\sum_{n=-2}^{7}e^{j\pi n/2} = e^{-j2\pi/2}\sum_{n=0}^{9}e^{j\pi n/2} = -(1+j).$$

(c) The desired sum is

$$\sum_{n=0}^{\infty}(1/2)^ne^{j\pi n/2} = \frac{1}{1-(1/2)e^{j\pi/2}} = \frac{4}{5} + j\frac{2}{5}.$$

(d) The desired sum is

$$\sum_{n=2}^{\infty}(1/2)^ne^{j\pi n/2} = (1/2)^2e^{j\pi 2/2}\sum_{n=0}^{\infty}(1/2)^ne^{j\pi n/2} = -\frac{1}{4}\left[\frac{4}{5} + j\frac{2}{5}\right].$$

(e) The desired sum is

$$\sum_{n=0}^{9}\cos(\pi n/2) = \frac{1}{2}\sum_{n=0}^{9}e^{j\pi n/2} + \frac{1}{2}\sum_{n=0}^{9}e^{-j\pi n/2} = \frac{1}{2}(1+j) + \frac{1}{2}(1-j) = 1.$$

(f) The desired sum is

$$\begin{aligned}\sum_{n=0}^{\infty}(1/2)^n\cos(\pi n/2) &= \frac{1}{2}\sum_{n=0}^{\infty}(1/2)^ne^{j\pi n/2} + \frac{1}{2}\sum_{n=0}^{\infty}(1/2)^ne^{-j\pi n/2} \\ &= \frac{4}{10} + j\frac{2}{10} + \frac{4}{10} - j\frac{2}{10} = \frac{4}{5}.\end{aligned}$$

1.56. (a) The desired integral is

$$\int_0^4 e^{j\pi t/2}dt = \left.\frac{e^{\pi t/2}}{j\pi/2}\right|_0^4 = 0.$$

(b) The desired integral is

$$\int_0^6 e^{j\pi t/2}dt = \left.\frac{e^{\pi t/2}}{j\pi/2}\right|_0^6 = (2/j\pi)[e^{j3\pi} - 1] = \frac{4j}{\pi}.$$

(c) The desired integral is

$$\int_2^8 e^{j\pi t/2}dt = \left.\frac{e^{\pi t/2}}{j\pi/2}\right|_2^8 = (2/j\pi)[e^{j4\pi} - e^{j\pi}] = -\frac{4j}{\pi}.$$

(d) The desired integral is

$$\int_0^{\infty} e^{-(1+j)t}dt = \left.\frac{e^{-(1+j)t}}{-(1+j)}\right|_0^{\infty} = \frac{1}{1+j} = \frac{1-j}{2}.$$

(e) The desired integral is

$$\int_0^\infty e^{-t}\cos(t)dt = \int_0^\infty \left[\frac{e^{-(1+j)t}+e^{-(1-j)t}}{2}\right]dt = \frac{1/2}{1+j}+\frac{1/2}{1-j} = \frac{1}{2}.$$

(f) The desired integral is

$$\int_0^\infty e^{-2t}\sin(3t)dt = \int_0^\infty \left[\frac{e^{-(2-3j)t}-e^{-(2+3j)t}}{2j}\right]dt = \frac{1/2j}{2-3j}+\frac{1/2j}{2+3j} = \frac{3}{13}.$$

Chapter 2 Answers

2.1. (a) We know that

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \qquad \text{(S2.1-1)}$$

The signals $x[n]$ and $h[n]$ are as shown in Figure S2.1.

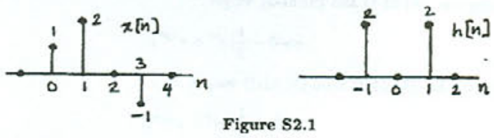

Figure S2.1

From this figure, we can easily see that the above convolution sum reduces to

$$\begin{aligned} y_1[n] &= h[-1]x[n+1] + h[1]x[n-1] \\ &= 2x[n+1] + 2x[n-1] \end{aligned}$$

This gives

$$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

(b) We know that

$$y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n+2-k]$$

Comparing with eq. (S2.1-1), we see that

$$y_2[n] = y_1[n+2]$$

(c) We may rewrite eq. (S2.1-1) as

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Similarly, we may write

$$y_3[n] = x[n] * h[n+2] = \sum_{k=-\infty}^{\infty} x[k]h[n+2-k]$$

Comparing this with eq. (S2.1), we see that

$$y_3[n] = y_1[n+2]$$

2.2. Using the given definition for the signal $h[n]$, we may write

$$h[k] = \left(\frac{1}{2}\right)^{k-1}\{u[k+3] - u[k-10]\}$$

The signal $h[k]$ is non zero only in the range $-3 \le k \le 9$. From this we know that the signal $h[-k]$ is non zero only in the range $-9 \le k \le 3$. If we now shift the signal $h[-k]$ by n to the right, then the resultant signal $h[n-k]$ will be non zero in the range $(n-9) \le k \le (n+3)$. Therefore,

$$A = n-9, \qquad B = n+3$$

2.3. Let us define the signals

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

and

$$h_1[n] = u[n].$$

We note that

$$x[n] = x_1[n-2] \quad \text{and} \quad h[n] = h_1[n+2]$$

Now,

$$\begin{aligned} y[n] &= x[n] * h[n] = x_1[n-2] * h_1[n+2] \\ &= \sum_{k=-\infty}^{\infty} x_1[k-2]h_1[n-k+2] \end{aligned}$$

By replacing k with $m+2$ in the abovr summation, we obtain

$$y[n] = \sum_{m=-\infty}^{\infty} x_1[m]h_1[n-m] = x_1[n] * h_1[n]$$

Using the results of Example 2.1 in the text book, we may write

$$y[n] = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right]u[n]$$

2.4. We know that

$$y[n] = x[n] * h[n] = \sum_{k=\infty}^{\infty} x[k]h[n-k]$$

The signals $x[n]$ and $y[n]$ are as shown in Figure S2.4. From this figure, we see that the above summation reduces to

$$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$$

This gives

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & \text{otherwise} \end{cases}$$

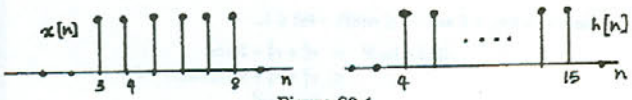

Figure S2.4

2.5. The signal $y[n]$ is

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

In this case, this summation reduces to

$$y[n] = \sum_{k=0}^{9} x[k]h[n-k] = \sum_{k=0}^{9} h[n-k]$$

From this it is clear that $y[n]$ is a summation of shifted replicas of $h[n]$. Since the last replica will begin at $n=9$ and $h[n]$ is zero for $n > N$, $y[n]$ is zero for $n > N+9$. Using this and the fact that $y[14] = 0$, we may conclude that N can *at most* be 4. Furthermore, since $y[4] = 5$, we can conclude that $h[n]$ has *at least* 5 non-zero points. The only value of N which satifies both these conditions is 4.

2.6. From the given information, we have:

$$\begin{aligned} y[n] &= x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^{-k} u[-k-1]u[n-k-1] \\ &= \sum_{k=-\infty}^{-1} \left(\frac{1}{3}\right)^{-k} u[n-k-1] \\ &= \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{k} u[n+k-1] \end{aligned}$$

Replacing k by $p-1$,

$$y[n] = \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p+1} u[n+p] \qquad \text{(S2.6-1)}$$

For $n \ge 0$ the above equation reduces to,

$$y[n] = \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p+1} = \frac{1}{3}\frac{1}{1-\frac{1}{3}} = \frac{1}{2}.$$