SEOUL NATIONAL UNIVERSITY
DEPARTMENT OF MECHANICAL AND AEROSPACE ENGINEERING

CONTROL SYSTEM THEORY **Fall 2010**

HW#3 **Assigned: October 6 (We)**
Due: October 18 (Mo)

Consider a 2 DOF bicycle Model as follows:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \dfrac{2\cdot F_{tyf} + 2\cdot F_{tyr}}{m\cdot v_x} - \gamma \\ \dfrac{2\cdot l_f \cdot F_{tyf} - 2\cdot l_r \cdot F_{tyr} + \sum_{i=1}^{4} M_{tzi}}{I_z} \end{bmatrix} \qquad \text{--Eq.1}$$

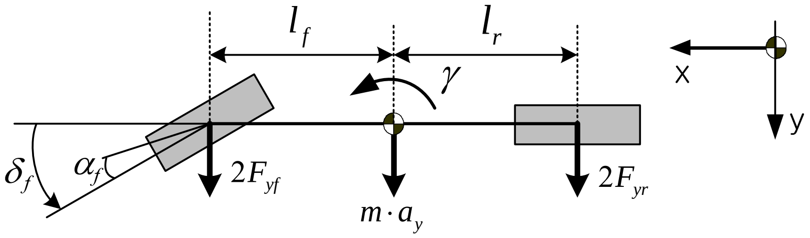

Fig.1 2-DOF Bicycle Model

Vehicle parameters of the 2 DOF bicycle model are listed in Table.1.

Table.1 Vehicle Parameters

	Symbol	Value		Symbol	Value
Vehicle mass	m	2450 kg	Length from C.G. to the rear wheel axis	l_r	$L - l_f$
Moment of inertia about the yaw axis	I_z	4331.6 kgm^2	Front Cornering Stiffness	C_f	73305 N/rad
Vehicle Length	L	2.85 m	Rear Cornering Stiffness	C_r	58850 N/rad
Length from C.G. to the front wheel axis	l_f	1.070 m	Friction Coefficient	μ	0.85

In Eq.1, lateral tire forces at front and rear wheels can be calculated using a non-linear function as follows:

▪ Lateral Tire Force

$$F_{tyf} = \frac{2}{\pi}\cdot\mu\cdot F_{zf}\cdot\tan^{-1}\left(\frac{\pi}{2\cdot\mu\cdot F_{zf}}\cdot C_f\cdot\alpha_f\right)$$
$$F_{tyr} = \frac{2}{\pi}\cdot\mu\cdot F_{zr}\cdot\tan^{-1}\left(\frac{\pi}{2\cdot\mu\cdot F_{zr}}\cdot C_r\cdot\alpha_r\right) \qquad \text{--Eq.2}$$

▪ Vertical Tire Force

$$2\cdot F_{zf} = \frac{m\cdot l_r}{l_f + l_r}\cdot g, \qquad 2\cdot F_{zr} = \frac{m\cdot l_f}{l_f + l_r}\cdot g$$

- Slip Angle

$$\alpha_f = \delta_f - \frac{v_y + l_f \cdot \gamma}{v_x} \qquad \alpha_r = -\frac{v_y - l_r \cdot \gamma}{v_x}$$

Also, a self aligning moment, M_{tzi}, at each wheel is given as 2-D Look-Up Table.

Table.2 Self Aligning Moment

Self Aligning Moment [Nm]		Vertical Tire Force [N]		
		2500 N	4100 N	5800 N
Slip Angle [deg]	-10 °	-6.2133 Nm	-20.86 Nm	-44 Nm
	-9 °	-7.7467 Nm	-25.53 Nm	-53 Nm
	-8 °	-9.8059 Nm	-31.50 Nm	-64.17 Nm
	-7 °	-12.2 Nm	-38.17 Nm	-76.32 Nm
	-6 °	-15.7 Nm	-47.02 Nm	-91.47 Nm
	-5 °	-19.44 Nm	-55.2 Nm	-103.8 Nm
	-4 °	-22.68 Nm	-61.22 Nm	-111.66 Nm
	-3 °	-24.61 Nm	-62.62 Nm	-110.1 Nm
	-2 °	-20.5 Nm	-49.9 Nm	-85.7 Nm
	-1 °	-13.33 Nm	-30.75 Nm	-51.25 Nm
	0 °	0 Nm	0 Nm	0 Nm
	1 °	13.33 Nm	30.75 Nm	51.25 Nm
	2 °	20.5 Nm	49.9 Nm	85.7 Nm
	3 °	24.61 Nm	62.62 Nm	110.1 Nm
	4 °	22.68 Nm	61.22 Nm	111.66 Nm
	5 °	19.44 Nm	55.2 Nm	103.8 Nm
	6 °	15.7 Nm	47.02 Nm	91.47 Nm
	7 °	12.2 Nm	38.17 Nm	76.32 Nm
	8 °	9.8059 Nm	31.50 Nm	64.17 Nm
	9 °	7.7467 Nm	25.53 Nm	53 Nm
	10 °	6.2133 Nm	20.86 Nm	44 Nm

Using the above lateral tire forces and self aligning moments, simulation study for the non-linear bicycle model can be conducted.

HW#3

1. Using the non-linear bicycle model, analyze the vehicle stability based on phase-plane method. The phase plane can be obtained from the state trajectory when a certain initial state is given. The vehicle simulations for phase plane analysis should be conducted under the following conditions:

- Vehicle speed is 100 km/h.
- Initial body slip angle should be simulated from -1.5 rad to 1.5 rad.
- Initial yaw rate should be simulated from -1.5 rad to 1.5 rad.
- Front steering angle, δ_f , is constant.

(1) Plot $\beta - \dot{\beta}$ phase plane trajectory and $\beta - \gamma$ phase plane trajectory at $\delta_f = 0\,\text{deg}$

(2) Plot $\beta - \dot{\beta}$ phase plane trajectory and $\beta - \gamma$ phase plane trajectory at $\delta_f = 3\,\text{deg}$

(3) Plot $\beta - \dot{\beta}$ phase plane trajectory and $\beta - \gamma$ phase plane trajectory at $\delta_f = 6\,\text{deg}$

2. Using the non-linear bicycle model and the front steering angle maneuver as shown in Fig.3, simulate the vehicle behaviors. The vehicle simulations should be conducted under the following conditions:

- Vehicle speed is 100 km/h.
- Initial states, $\begin{bmatrix}\beta & \gamma\end{bmatrix}^T$, are set to be $\begin{bmatrix}0 & 0\end{bmatrix}^T$.

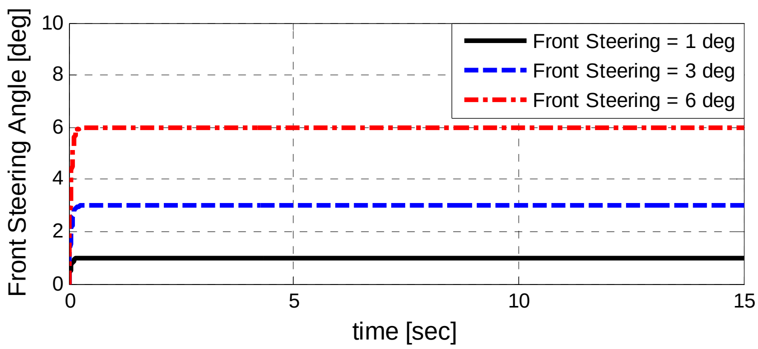

Fig.2 Front steering angle [deg]

(1) Plot body slip angle and yaw rate in time domain.

(2) Plot vehicle trajectory in global X-Y plane.

(3) Plot vehicle trajectory in $\beta - \gamma$ and $\beta - \dot{\beta}$ phase planes.