

AE545 Homework Problem Set #1 (Solution)Problem #1

(a) $\frac{\Delta p}{p_\infty}$ across the disk is given by blade element theory

$$p_2 - p_1 = \frac{\rho}{2} w^2 = \frac{\rho}{2} 4v^2 = 2 \rho v^2$$

$$v = \frac{T}{2\rho A} \quad \text{thus} \quad \Delta p = p_2 - p_1 = 2\rho \frac{T}{3\rho A} = \frac{T}{A}$$

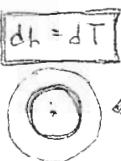
$$\frac{\Delta p}{p_\infty} = \frac{1}{\rho_\infty} \frac{T}{A} = \frac{3000}{2116(804)} = 0.00176$$

(b) Induced velocity far below the rotor is given by

$$w = 2v = 2 \sqrt{\frac{T}{2\rho A}} = 2 \sqrt{\frac{3000}{2(0.00238)(804)}} = 56 \text{ ft/sec.}$$

$$(c) C_T = \frac{T}{\rho \pi R^2 (\sqrt{2}R)^2} = \frac{3000}{0.00238(804) 700^2} = 0.0032$$

(d) The local lift coefficient is given by combining momentum and blade element theory for an annular portion of the disk [extended blade element theory]



$$\int v(r) 2\pi r dr 2v(r) = dT = N_b \frac{\rho}{2} (\sqrt{2}r) C_{le} dr = dL$$

$$C_{le} = \frac{8\pi v^2 r}{N_b (\sqrt{2}r)^2 c} = \frac{8\pi v^2 R \frac{r}{R}}{N_b (\sqrt{2}R)^2 c (\frac{r}{R})^2} = \frac{8v^2}{(\sqrt{2}R)^2 \frac{(CN_b)}{R\pi}}$$

$$\frac{R}{r} = \frac{8v^2}{(\sqrt{2}R)^2 C_{le} r}$$

$$C_{le}(r=0.5) = \frac{8v^2}{C_{le} (\sqrt{2}R)^2} \frac{R}{r} = \frac{8(28)^2 \frac{R}{2}}{0.0398(700)^2} = 0.64$$

(e) Local angle of attack

$$C_L = \alpha \left(\theta - \frac{v}{\rho R} \right)$$

$$\theta = \frac{\ell e}{a} + \frac{v}{\rho R} = \frac{C_L}{a} + \frac{v}{\rho R} \frac{R}{r} = \frac{28(2)}{700} + \frac{64}{6} = 0.1867$$

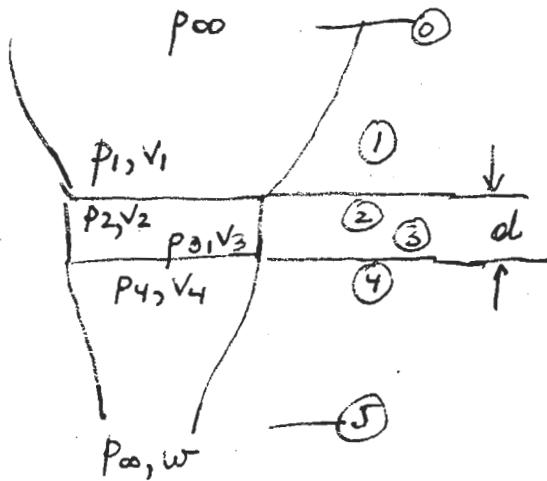
or 10.7°

(f) $\frac{C_{p0}}{C_{pi}} = \frac{\sqrt{C_{d0}} \sqrt{2}}{8 C_T^{3/2}} = \frac{0.0398(0.01)}{8} \frac{\sqrt{2}}{(0.0032)^{3/2}} = 0.39$

500 SHEETS FILLER 5 SQUARE
500 SHEETS EYE-EASE 5 SQUARE
100 SHEETS EYE-EASE 5 SQUARE
200 SHEETS EYE-EASE 5 SQUARE
100 RECYCLED WHITE 5 SQUARE
42-392 200 RECYCLED WHITE 5 SQUARE
42-391
42-392
42-393
42-394
42-395
42-396
42-397
42-398
42-399
42-400
42-401
42-402
42-403
42-404
42-405
42-406
42-407
42-408
42-409
42-410
42-411
42-412
42-413
42-414
42-415
42-416
42-417
42-418
42-419
42-420
42-421
42-422
42-423
42-424
42-425
42-426
42-427
42-428
42-429
42-430
42-431
42-432
42-433
42-434
42-435
42-436
42-437
42-438
42-439
42-440
42-441
42-442
42-443
42-444
42-445
42-446
42-447
42-448
42-449
42-450
42-451
42-452
42-453
42-454
42-455
42-456
42-457
42-458
42-459
42-460
42-461
42-462
42-463
42-464
42-465
42-466
42-467
42-468
42-469
42-470
42-471
42-472
42-473
42-474
42-475
42-476
42-477
42-478
42-479
42-480
42-481
42-482
42-483
42-484
42-485
42-486
42-487
42-488
42-489
42-490
42-491
42-492
42-493
42-494
42-495
42-496
42-497
42-498
42-499
42-500
42-501
42-502
42-503
42-504
42-505
42-506
42-507
42-508
42-509
42-510
42-511
42-512
42-513
42-514
42-515
42-516
42-517
42-518
42-519
42-520
42-521
42-522
42-523
42-524
42-525
42-526
42-527
42-528
42-529
42-530
42-531
42-532
42-533
42-534
42-535
42-536
42-537
42-538
42-539
42-540
42-541
42-542
42-543
42-544
42-545
42-546
42-547
42-548
42-549
42-550
42-551
42-552
42-553
42-554
42-555
42-556
42-557
42-558
42-559
42-560
42-561
42-562
42-563
42-564
42-565
42-566
42-567
42-568
42-569
42-570
42-571
42-572
42-573
42-574
42-575
42-576
42-577
42-578
42-579
42-580
42-581
42-582
42-583
42-584
42-585
42-586
42-587
42-588
42-589
42-590
42-591
42-592
42-593
42-594
42-595
42-596
42-597
42-598
42-599
42-600
42-601
42-602
42-603
42-604
42-605
42-606
42-607
42-608
42-609
42-610
42-611
42-612
42-613
42-614
42-615
42-616
42-617
42-618
42-619
42-620
42-621
42-622
42-623
42-624
42-625
42-626
42-627
42-628
42-629
42-630
42-631
42-632
42-633
42-634
42-635
42-636
42-637
42-638
42-639
42-640
42-641
42-642
42-643
42-644
42-645
42-646
42-647
42-648
42-649
42-650
42-651
42-652
42-653
42-654
42-655
42-656
42-657
42-658
42-659
42-660
42-661
42-662
42-663
42-664
42-665
42-666
42-667
42-668
42-669
42-670
42-671
42-672
42-673
42-674
42-675
42-676
42-677
42-678
42-679
42-680
42-681
42-682
42-683
42-684
42-685
42-686
42-687
42-688
42-689
42-690
42-691
42-692
42-693
42-694
42-695
42-696
42-697
42-698
42-699
42-700
42-701
42-702
42-703
42-704
42-705
42-706
42-707
42-708
42-709
42-710
42-711
42-712
42-713
42-714
42-715
42-716
42-717
42-718
42-719
42-720
42-721
42-722
42-723
42-724
42-725
42-726
42-727
42-728
42-729
42-730
42-731
42-732
42-733
42-734
42-735
42-736
42-737
42-738
42-739
42-740
42-741
42-742
42-743
42-744
42-745
42-746
42-747
42-748
42-749
42-750
42-751
42-752
42-753
42-754
42-755
42-756
42-757
42-758
42-759
42-760
42-761
42-762
42-763
42-764
42-765
42-766
42-767
42-768
42-769
42-770
42-771
42-772
42-773
42-774
42-775
42-776
42-777
42-778
42-779
42-780
42-781
42-782
42-783
42-784
42-785
42-786
42-787
42-788
42-789
42-790
42-791
42-792
42-793
42-794
42-795
42-796
42-797
42-798
42-799
42-800
42-801
42-802
42-803
42-804
42-805
42-806
42-807
42-808
42-809
42-810
42-811
42-812
42-813
42-814
42-815
42-816
42-817
42-818
42-819
42-820
42-821
42-822
42-823
42-824
42-825
42-826
42-827
42-828
42-829
42-830
42-831
42-832
42-833
42-834
42-835
42-836
42-837
42-838
42-839
42-840
42-841
42-842
42-843
42-844
42-845
42-846
42-847
42-848
42-849
42-850
42-851
42-852
42-853
42-854
42-855
42-856
42-857
42-858
42-859
42-860
42-861
42-862
42-863
42-864
42-865
42-866
42-867
42-868
42-869
42-870
42-871
42-872
42-873
42-874
42-875
42-876
42-877
42-878
42-879
42-880
42-881
42-882
42-883
42-884
42-885
42-886
42-887
42-888
42-889
42-890
42-891
42-892
42-893
42-894
42-895
42-896
42-897
42-898
42-899
42-900
42-901
42-902
42-903
42-904
42-905
42-906
42-907
42-908
42-909
42-910
42-911
42-912
42-913
42-914
42-915
42-916
42-917
42-918
42-919
42-920
42-921
42-922
42-923
42-924
42-925
42-926
42-927
42-928
42-929
42-930
42-931
42-932
42-933
42-934
42-935
42-936
42-937
42-938
42-939
42-940
42-941
42-942
42-943
42-944
42-945
42-946
42-947
42-948
42-949
42-950
42-951
42-952
42-953
42-954
42-955
42-956
42-957
42-958
42-959
42-960
42-961
42-962
42-963
42-964
42-965
42-966
42-967
42-968
42-969
42-970
42-971
42-972
42-973
42-974
42-975
42-976
42-977
42-978
42-979
42-980
42-981
42-982
42-983
42-984
42-985
42-986
42-987
42-988
42-989
42-990
42-991
42-992
42-993
42-994
42-995
42-996
42-997
42-998
42-999
42-1000

Homework Problem Set #1 - Prob #2Sample Solution

- (2) There are a number of possible approaches to solve this problem. The first one is illustrated below.



Assume d is small.
Combine two rotors in one equivalent rotor

$$T = 6500 \text{ lb}$$

Instead of 2-blades \rightarrow has 4 blades

C = solidity of equivalent rotor
~~is twice~~ ~~solidity of each rotor~~

Since there is no wake contraction between the rotors, for incompressible flow conservation of mass yields

$$\rho v_1 = \rho v_3 \quad \text{thus} \quad v_1 = v_3 \quad \text{and} \quad p_2 = p_3$$

there is a constant induced velocity through the dual, coaxial rotor system. Thus a feasible model is to assume a single rotor with solidity equal to twice the solidity of each rotor, which is essentially equivalent to requiring $d=0$.

Then we can assume a fully developed momentum wake, below the "equivalent" rotor. The inflow is given by

$$\lambda = \frac{V}{2R} = \frac{1}{2R} \sqrt{\frac{T}{2\rho(\pi R^2)}}$$

$$(1) \quad \lambda = \frac{C}{2\pi R} \sqrt{\frac{T}{2\rho}}$$

where T is the total thrust ($T = 6500 \text{ lb}$).

Having ideal twist, guarantees uniform inflow over the disc.

$$\Omega = 2\pi n = 2\pi \left(\frac{300}{60} \right) = 10\pi \text{ rad/sec}$$

2

$$\lambda = \frac{1}{31.4(15.417)} \sqrt{\frac{6500}{2(0.00238)(3.14)(15.417)^2}} = \underline{0.0884} \quad (0.08836)$$

Using blade element theory the thrust of the "equivalent" rotor can be calculated, assuming small angles (and $x = r/R$)

$$T \approx b \int_0^R dr = \frac{1}{2} \rho abc \alpha c^2 \int_0^R r^2 \left[\frac{\theta_t}{(r/R)} - \frac{v}{\alpha r} \right] dr =$$

$$= \frac{1}{2} \rho abc \alpha c^2 R^3 \int_0^1 x^2 \left[\frac{\theta_t}{x} - \frac{v}{\alpha R x} \right] dx = \frac{1}{2} \rho abc \alpha c^2 R^3 \frac{1}{2} (\theta_t - \lambda)$$

$$C_T = \frac{T}{\rho \pi R^2 (\sqrt{2} R)^2} = \frac{\sigma a}{4} (\theta_t - \lambda) \quad (2)$$

where $\sigma = \frac{bc}{TR}$ = solidity ratio of the "equivalent" rotor, based upon the total number of blades

$$\text{From Eq(2)} \quad \theta_t = \underbrace{\frac{4C_T}{\sigma\sigma}}_{\alpha\sigma} + \lambda. \quad (3)$$

$$C_T = \frac{T}{\rho (\pi R^2) (\sqrt{2} R)^2} = \frac{6500}{(0.00238)(3.14)(15.417)^2 (31.4)^2 (15.417)^2} = \underline{0.0156}$$

$$\sigma = \frac{4(25)}{3.14(185)} = 0.172$$

$$\theta_t = 0.08836 + \frac{4(0.0156)}{5.73(0.172)} = 0.1517 = \underline{8.69^\circ}$$

i.e. both rotors have the same pitch setting.

The torque can be also calculated from blade element theory for the "equivalent" rotor

$$Q = b \int_0^R r dD' = b \int_0^R (\varphi dL + dD_o) dr =$$

$$= b \int_0^R r \left\{ \frac{1}{2} \rho abc \alpha^2 r^2 \left(\frac{\theta_t}{x} - \frac{v}{\alpha r} \right) \frac{v}{\alpha r} + \frac{1}{2} \rho C_{D_o} c \alpha^2 r^2 \right\} dr$$

$$= \frac{1}{2} \rho abc \alpha^2 \int_0^1 \left[\left(\frac{\theta_t x^2}{x} - \frac{v x^2}{\alpha R x} \right) R^4 \frac{v}{\alpha R} dx + \frac{1}{2} \rho C_{D_o} c b \alpha^2 R^4 \right]$$

$$= \frac{1}{2} \rho abc \alpha^2 R^4 \frac{1}{2} (\theta_t - \lambda) + \frac{1}{8} \rho C_{D_o} c b \alpha^2 R^4$$

3

$$C_Q = C_P = \frac{Q}{\rho \pi R^2 (\sqrt{2}R)^2 R} = \frac{1}{4} \alpha C(\theta_t - \lambda) \lambda + \frac{1}{8} C_{D0} C \quad (4)$$

using Eq(2)

$$C_Q = C_P = \lambda C_T + \frac{1}{8} C_{D0} \quad (5)$$

The power required

$$P = C_P \rho \pi R^2 \left(\frac{\sqrt{2}R}{550} \right)^3 = \frac{598 \text{ HP}}{550}$$

$$C_P = (0.08336)(0.0156) + \frac{1}{8}(0.172)(0.012) = 0.001558$$

- ② If there is streamline contraction, the inflow velocity and the inflow angle on the second rotor have to be higher than on the first rotor, where the second rotor denotes the lower (or bottom) rotor. Assuming equal thrust

$$T_1 = T_2 = T/2$$

the "effective" angles of attack have to be equal

$$\theta_{t1} - \frac{v_1}{\sqrt{2}R} = \theta_{t2} - \frac{v_2}{\sqrt{2}R}$$

if $v_2 > v_1$ due to slipstream contraction

$$\theta_{t2} > \theta_{t1}$$

the lower rotor blades require a higher pitch setting.

An alternative approach to the solution of this problem is also provided below for the sake of completeness or clarity.

Applying momentum theory for station ③ above the rotors and station ④ below the coaxial rotors we have

$$\bar{T}_{OT} = \bar{T}_1 + \bar{T}_2 = 2\bar{T} = \rho A v w \quad (6)$$

(4)

Since $v_1 = v_3$ and $p_2 = p_3$
 applying Bernoulli's law between $\textcircled{0} \rightarrow \textcircled{1}$ yields

$$p_0 + \frac{1}{2} \rho v^2 = p_1 + \frac{1}{2} \rho v^2 \quad (7)$$

applying Bernoulli's law between $\textcircled{4} \rightarrow \textcircled{5}$ yields

$$p_4 + \frac{1}{2} \rho v^2 = p_0 + \frac{1}{2} \rho w^2$$

$$\text{or } p_4 - p_0 = \frac{1}{2} \rho w^2 - \frac{1}{2} \rho v^2 \quad (8)$$

Combining (7) & (8)

$$p_4 - p_1 - \frac{1}{2} \cancel{\rho v^2} = \frac{1}{2} \rho w^2 - \cancel{\frac{1}{2} \rho v^2}$$

$$p_4 - p_1 = \frac{\rho w^2}{2} \quad (9)$$

and $(p_4 - p_1) A = 2 T = \frac{\rho w^2}{2} A = \rho A v w$

when we have used Eq(6)

thus $w = 2v \quad (10)$

and using Eq(10) $2 T = \rho A v w = 2 \rho A v^2$

$$v = \sqrt{\frac{T}{\rho A}} = \sqrt{2} \sqrt{\frac{T}{2 \rho A}} = \sqrt{2} v_H \quad (11)$$

where v_H is the hover inflow for an isolated rotor
 producing 3250 lb of thrust

from eq(11) $v = 42.794 \text{ ft/sec}$

$$\lambda = \frac{v}{\Omega R} = 0.08836$$

which is the same inflow as that given on the top of
 page 2.

The rest of the calculation can be carried out in a
 similar manner to the previous case.