재료공학원리 Homework

9장 예제 문제 풀이

1.

EXAMPLE PROBLEM 11.1

Lever Rule Derivation

Derive the lever rule.

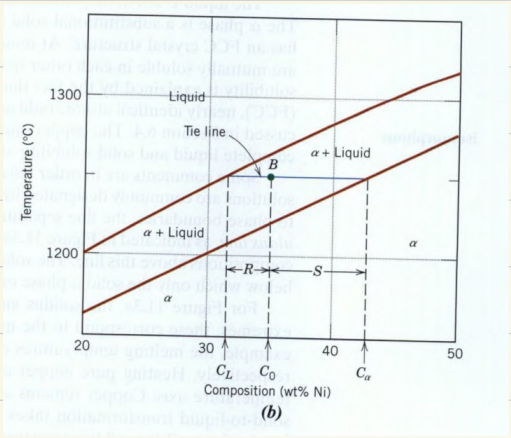

Figure 11.3 (*a*) The copper–nickel phase diagram. (*b*) A portion of the copper–nickel phase diagram for which compositions and phase amounts are determined at point *B*. (Adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash, Editor, 1991. Reprinted by permission of ASM International, Materials Park, OH.)

점 B의 composition과 temperature에서는 α phase와 L phase가 존재한다. 다른 상은 존재하지 않으므로, 다음이 성립한다.

$W_\alpha + W_L = 1 \cdots ①$

Ni의 질량비는 α에서 C_α, L에서 C_L이다. 전체에선 C_0이므로, 다음이 성립한다.

$C_\alpha W_\alpha + C_L W_L = C_0 \cdots ②$

$C_L \times ① - ②$: $(C_L - C_\alpha) W_\alpha = C_L - C_0$

$$\therefore W_\alpha = \frac{C_0 - C_L}{C_\alpha - C_L} = \boxed{\frac{R}{R+S}}$$

$C_\alpha \times ① - ②$: $(C_\alpha - C_L) W_L = C_\alpha - C_0$

$$\therefore W_L = \frac{C_\alpha - C_0}{C_\alpha - C_L} = \boxed{\frac{S}{R+S}}$$

2.

EXAMPLE PROBLEM 11.2

Determination of Phases Present and Computation of Phase Compositions

For a 40 wt% Sn–60 wt% Pb alloy at 150°C, **(a)** what phase(s) is (are) present? **(b)** What is (are) the composition(s) of the phase(s)?

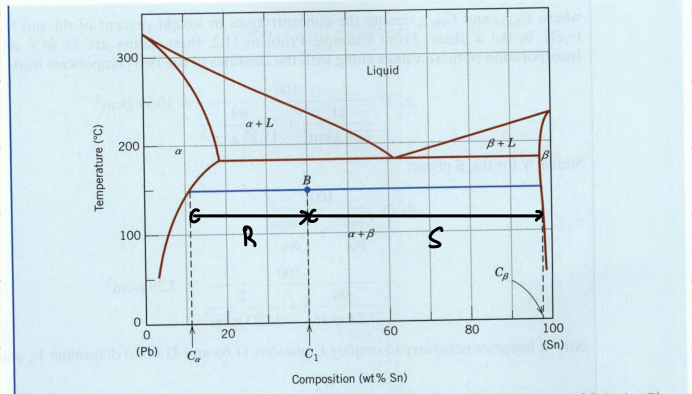

Figure 11.8 The lead–tin phase diagram. For a 40 wt% Sn–60 wt% Pb alloy at 150°C (point *B*), phase compositions and relative amounts are computed in Example Problems 11.2 and 11.3.

(a) α phase와 β phase가 존재한다.

(b) α phase의 composition = C_α이므로

α : 11 wt% Sn – 89 wt% Pb

β phase의 composition = C_β이므로

β : 99 wt% Sn – Pb wt% Pb

3.

EXAMPLE PROBLEM 11.3

Relative Phase Amount Determinations—Mass and Volume Fractions

For the lead–tin alloy in Example Problem 11.2, calculate the relative amount of each phase present in terms of **(a)** mass fraction and **(b)** volume fraction. At 150°C, take the densities of Pb and Sn to be 11.23 and 7.24 g/cm^3, respectively.

(a)

$$W_\alpha = \frac{S}{R+S} = \frac{99-40}{99-11} = \underline{0.67}$$

$$W_\beta = \frac{R}{R+S} = \frac{40-11}{99-11} = \underline{0.33}$$

(lever rule)

(b)

$$\rho_\alpha = \frac{m_{Sn} + m_{Pb}}{V_{Sn} + V_{Pb}}$$

m : mass, V: volume, ρ : density

$$V_{Sn} = \frac{m_{Sn}}{\rho_{Sn}}$$

$$\text{또한} \quad C_{Sn,\alpha} = \frac{m_{Sn}}{m_{Sn} + m_{Pb}} \quad \therefore m_{Sn} = C_{Sn,\alpha}(m_{Sn} + m_{Pb})$$

$$\therefore V_{Sn} = \frac{C_{Sn,\alpha}(m_{Sn} + m_{Pb})}{\rho_{Sn}}$$

같은 방법으로,

$$V_{Pb} = \frac{C_{Pb,\alpha}(m_{Sn} + m_{Pb})}{\rho_{Pb}}$$

$$\therefore V_{Sn} + V_{Pb} = (m_{Sn} + m_{Pb})\left(\frac{C_{Sn,\alpha}}{\rho_{Sn}} + \frac{C_{Pb,\alpha}}{\rho_{Pb}}\right)$$

$$\therefore \rho_\alpha = \frac{m_{Sn} + m_{Pb}}{V_{Sn} + V_{Pb}} = 1 \Big/ \left(\frac{C_{Sn,\alpha}}{\rho_{Sn}} + \frac{C_{Pb,\alpha}}{\rho_{Pb}}\right)$$

$$= 1 \Big/ \left(\frac{0.11}{7.24\ g\,cm^{-3}} + \frac{0.89}{11.23\ g\,cm^{-3}}\right)$$

$$= 10.59\ g\,cm^{-3}$$

$$\rho_\beta = 1 \Big/ \left(\frac{C_{Sn,\beta}}{\rho_{Sn}} + \frac{C_{Pb,\beta}}{\rho_{Pb}}\right)$$

$$= 1 \Big/ \left(\frac{0.98}{7.24\ g\,cm^{-3}} + \frac{0.02}{11.23\ g\,cm^{-3}}\right)$$

$$= 7.29\ g\,cm^{-3}$$

Volume fraction of α

$$= \frac{m_\alpha/\rho_\alpha}{m_\alpha/\rho_\alpha + m_\beta/\rho_\beta} = \frac{W_\alpha/\rho_\alpha}{W_\alpha/\rho_\alpha + W_\beta/\rho_\beta}$$

$$= \frac{0.67/10.59\ g\,cm^{-3}}{0.67/10.59\ g\,cm^{-3} + 0.33/7.29\ g\,cm^{-3}}$$

$$= 0.58$$

Volume fraction of β

$$= \frac{W_\beta/\rho_\beta}{W_\alpha/\rho_\alpha + W_\beta/\rho_\beta} = 0.42$$

4.

EXAMPLE PROBLEM 11.4

Determination of Relative Amounts of Ferrite, Cementite, and Pearlite Microconstituents

For a 99.65 wt% Fe–0.35 wt% C alloy at a temperature just below the eutectoid, determine the following:

(a) The fractions of total ferrite and cementite phases
(b) The fractions of the proeutectoid ferrite and pearlite
(c) The fraction of eutectoid ferrite

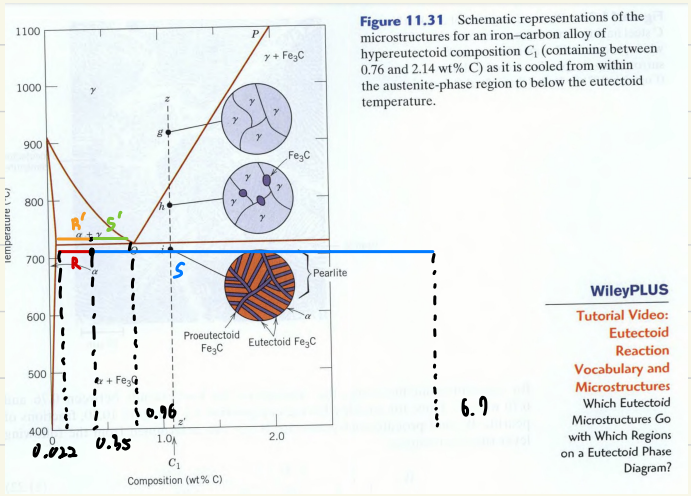

Figure 11.31 Schematic representations of the microstructures for an iron–carbon alloy of hypereutectoid composition C_1 (containing between 0.76 and 2.14 wt% C) as it is cooled from within the austenite-phase region to below the eutectoid temperature.

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Tutorial Video: Eutectoid Reaction Vocabulary and Microstructures
Which Eutectoid Microstructures Go with Which Regions on a Eutectoid Phase Diagram?

(a)

$C_0 = 0.35, \quad C_\alpha = 0.022, \quad C_{Fe_3C} = 6.7$

$$\therefore W_\alpha = \frac{S}{R+S} = \frac{6.7 - 0.35}{6.7 - 0.022} = 0.95$$

$$W_\beta = \frac{R}{R+S} = \frac{0.35 - 0.022}{6.7 - 0.022} = 0.05$$

(b)

Tie line just "above" eutectoid 일 때

$$W_{pearlite} = W_{\gamma} = \frac{R'}{R' + S'} = \frac{0.35 - 0.022}{0.76 - 0.022}$$

$$= \underline{0.44}$$

$$W_{proeutectoid\ \alpha} = \frac{S'}{R' + S'} = \frac{0.76 - 0.35}{0.76 - 0.022}$$

$$= \underline{0.56}$$

(c)

just above eutectoid

$$\xleftarrow[\text{proeutectoid } \alpha]{0.56} \times \xrightarrow[\gamma]{0.44}$$

$$\downarrow \quad \gamma \rightleftharpoons \alpha + Fe_3C$$

just below eutectoid

$$\xleftarrow[\text{proeutectoid } \alpha]{0.56} \times \underset{\substack{\text{eutec}\\-\text{toid}\\ \alpha}}{W_{a e}} \times \xrightarrow[Fe_3C]{0.05}$$

$W_{ae} = 1 - 0.56 - 0.05 = \underline{0.39}$