Homework #1 (Due: 10/4)

1. Problem 1.13 (Simon) in the text.

2. Problem 2.10 (Simon) in the text.

3. Show that

$$\begin{bmatrix} x-\overline{x} \\ z-\overline{z} \end{bmatrix}^T P_Y^{-1} \begin{bmatrix} x-\overline{x} \\ z-\overline{z} \end{bmatrix} - (z-\overline{z})^T P_z^{-1}(z-\overline{z}) = (x-E\{x \mid z\})^T P_{x|z}^{-1}(x-E\{x \mid z\})$$

   employing Eqs. (1.25)-(1.28) in the text (Lewis).

4. Show that

$$\left|P_Y\right| / \left|P_z\right| = \left|P_{x|z}\right|$$

   of Eq. (1.29) in the text (Lewis).

5. Prove the following theorem.
   Theorem: (Cramer-Rao Bound)
   If $\hat{X}$ is any unbiased estimate of a deterministic, scalar variable *X* based on measurement *Z*, then the covariance of the estimation error $\tilde{X} = X - \hat{X}$ is bounded by

$$P_{\tilde{X}} \geq J_F^{-1}$$

   where the Fisher information matrix is given by

$$J_F = E\left\{\left[\frac{\partial \ln f_{Z|X}(z \mid x)}{\partial X}\right]^2\right\} = -E\left\{\frac{\partial^2 \ln f_{Z|X}(z \mid x)}{\partial X^2}\right\}.$$

   It is assumed that $\dfrac{\partial f_{Z|X}}{\partial X}$ and $\dfrac{\partial^2 f_{Z|X}}{\partial X^2}$ exist and are absolutely integrable.